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Taxes on risky returns – an update

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Abstract

This paper surveys the theory on taxes on risky returns that originated from Domar and Musgrave (1944). Emphasis is given to the role of complete capital markets and on capital market imperfections arising from limited liability, moral hazard and adverse selection.

Keywords: risk taking, taxation, capital markets

JEL classification numbers: H22, H25

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# Introduction

There are many different economic opinions about how taxes affect risk-taking decisions and whether this is good or bad from a normative point of view. It is often asserted, for example, that too little venture capital is made available or that large corporations have lost their willingness to take risks. It is claimed that entrepreneurship is giving way to atrophy and our society is becoming a club for rent-income earners. Politicians often make presumably futile public appeals to investors, urging them to increase their appetite for risky investments. On the other side there are claims that some industries, and the financial industry in particular, take excessive risks, and that the severe financial crisis that started around 2007 had its deeper roots in reckless risk-taking. The government has a potentially important role in the allocation process that governs risk-taking. It provides the institutional framework and is able to intervene more directly, for instance, via taxes. This paper uses some building blocks from Buchholz and Konrad (2000) and further adds more recent results for a systematic exposition of the positive and normative effects of taxes on risky returns.\(^1\) We start this with a brief overview of what has been done and how we take this on board for our survey.

Early on, the literature challenged the claim that taxes on risky returns of investment reduced investors' affinity for risky investments and pushed them toward safer (fixed-interest) investments. Domar and Musgrave (1944), Mossin (1968), Stiglitz (1969), Allingham (1972), and others derived results that point in the opposite direction. Taxes on risky returns reduce the risk exposure of private investors. Such taxes lift the risks from the shoulders of private investors and move it toward the public sector. The private sector, left with fewer risk-taking than it would like to take could then widen the risk-taking activities. Advocates of this Domar-Musgrave view on the governmental role in risk-taking are, among others, Gordon (1985) and Sinn (1995). The argument strongly relies on the assumption that the risk born in the private sector is to a large extent idiosyncratic, and that the government can consolidate this risk within the collected sum of tax revenues on the risky returns of many investments by many different investors. Partial analysis of the effect of taxation on risky investment decisions abstains from asking what happens with the tax revenue. This simplification is crucial, but also makes the important underlying effects for this argument most transparent. Section 2 in the current survey revisits this literature and provides an overview of major comparative static results. It also discusses taxation under complete versus incomplete loss-offset provisions and the role of limited liability.

The Domar-Musgrave view on the effect of taxing risky returns on risk-taking and the normative appeal of such taxes in particular have been challenged by a number of authors, based on different grounds. Bulow and Summers (1984) and Konrad (1991) showed that the

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\(^1\) The survey has a partial view on a large topic, and so many aspects are not covered. For instance, taxation of risky returns also has intertemporal aspects. Starting with Gordon and Varian (1988), a government may attempt to smooth consumption across generations. A literature that parallels a discussion of the risky returns of firms or in corporate finance considers risky investment in human capital, starting with contributions by Eaton and Rosen (1980a,b). Schindler (2008) partially surveys this literature, highlights the taxation of risky returns in a labor market context and incorporates a savings decision.
incentive effect of such taxes on risk-taking cannot be sustained in a system with perfect and complete capital markets. For a clear and intuitive discussion from a lawyer’s perspective see also Weisbach (2004). The claim of this general equilibrium literature is that a tax on risk-taking is perfectly neutral: it is ineffective but mostly harmless, and to the extent that this is the case, the tax revenue that is generated by taxing the pure returns has a positive expected value, but is worthless – it has a market value of zero. Section 3 analyzes this argument rigorously within a general equilibrium framework. Using the logic first applied in Buchholz and Konrad (2000) we consider the problem as a strategic game and apply the concept of Nash equilibrium. The analysis shows that the neutrality of taxes on risk-taking emerges not only under conditions of complete capital markets with perfect competition in a Walrasian equilibrium, but even if the set of players is rather small. The analysis also considers the case of limited liability. It emerges that limited liability may cause multiple equilibria. One equilibrium is efficient. The other equilibrium is based on a strategic complementarity of excessive risk-taking and has strictly lower welfare.

From here, we must consider the fact that capital markets are not perfect or complete, leading to the question of whether this might be a means for rehabilitating the partial analytic results or the claims that the government could play a major role in consolidating risks through taxation. In fact, when the private sector just abstains from risk sharing for fully exogenous reasons, such a rehabilitation seems to work. However, an appropriate account for market incompleteness must take on board the reasons for incompleteness. This is not for reasons of consistency or elegance, but because the introduction of a tax interacts with the very reasons that may cause capital market incompleteness, and a neglect of this interaction can yield strongly misleading conclusions. A more recent literature focuses on the taxes on risk-taking in contexts in which capital markets are imperfect or incomplete for reasons of incomplete information. This literature is described in more detail in Section 4. The literature includes discussions of moral hazard (Buchholz and Konrad 2000; Keuschnigg and Nielsen 2003, 2004; and Hagen and Sannarnes 2007) and of adverse selection. It shows that private investors may voluntarily bear idiosyncratic risks that could be consolidated in a first-best world either in the capital market or within the sum of revenues from taxing risky returns. This risk-bearing is not a market failure, but it is a tool for partially overcoming the efficiency problems created by incomplete information. Bearing idiosyncratic risks can have an important function in the contractual arrangements in the private market. As it turns out, in such a context the private sector may react to a tax on risky returns by widening the amount of idiosyncratic risks that are taken by private individuals. The Domar-Musgrave effect is recovered in this context, but the normative interpretation is very different. The original Domar Musgrave literature in section 2 argues that unconsolidated idiosyncratic risk is a market failure and taxes on risky returns is a way of coping with it. This view is challenged by the analysis in the papers on moral hazard: unconsolidated idiosyncratic risk serves an important purpose as an incentive device, and its taxation is dysfunctional. The private markets may have the scope to perfectly counterbalance the negative side effects of these taxes, but only under rather special circumstances. Only under these special conditions
is the tax inconsequential and harmless. Where it has consequences, the tax is harmful. Section 4 also discusses other information reasons for why the private market may be left with undiversified idiosyncratic risks, surveying further contributions on this matter.

Section 5 offers an assessment of the theoretical arguments and policy conclusions about the role of taxes on risky returns.

2 The partial analytical portfolio decision

Consider a typical investor\(^2\) who decides on the allocation of his initial assets, of value \(a\). He has two opportunities for investment. One of these is the safe investment, which pays out a revenue of \((1 + r)\) units for each unit invested. In general we assume \(r \geq 0\). The borderline case \(r = 0\) plays an important role as a benchmark. The other investment opportunity is a risky investment that pays out a revenue of \((1 + \tilde{z})\) units for every unit invested, whereby \(\tilde{z}\) is a random variable. A risk-averse investor would not be faced with a real decision problem for any case where \(E\tilde{z} \leq r\). He would invest all of his initial assets in the safe alternative. Likewise, if all of the possible values for \(\tilde{z}\) were larger than \(r\), the decision problem would not be very interesting. Every investor who values more income over less income would choose to invest all of his capital in the risky investment opportunity. Interesting problems emerge if the expected value of \(\tilde{z}\) is greater than \(r\), i.e., \(E\tilde{z} > r\), and if \(\tilde{z}\) can take values greater and less than \(r\) with a positive probability. A special case that has a simple graphical representation would be useful here. For this we sometimes restrict \(\tilde{z}\) to two possible outcomes, \(z_1 > r\) and \(z_2 < r\), that occur with the respective probabilities \(\pi_1\) and \(\pi_2 = 1 - \pi_1\), and with \(E(\tilde{z}) = \pi_1 z_1 + \pi_2 z_2 > r\). This case can be expressed graphically. The random variable \(\tilde{z}\) has several possible interpretations. It can be interpreted as a single risky investment opportunity. Alternatively, it can also be interpreted as a share in the market portfolio of risky assets, with \(\tilde{z}\) as the risky return on an investment in the “market portfolio” per unit of investment.\(^3\) The parameter restrictions on \(\tilde{z}\) which we just discussed, follow from capital market equilibrium analysis in this latter case. Finally, let \(m\) be the amount of capital the investor invests in the risky option. Thus, his final net wealth in the absence of taxation is equal to

\[
(a - m)(1 + r) + m(1 + \tilde{z}) = a(1 + r) + m(\tilde{z} - r). \tag{1}
\]

Suppose now that the government levies a proportional income tax with a rate of \(\tau\)

\(^2\)The presentation follows lines of reasoning in Mossin (1986), Stiglitz (1969), Atkinson and Stiglitz (1980) and Sandmo (1985). It focuses on the question of how a tax influences the application of a given amount invested. The savings decision is not considered. This intertemporal allocation decision is also affected by the capital income tax.

\(^3\)Analysis of the situation of an investor who can invest in multiple unsafe investments alongside safe investments leads to the same (Domar-Musgrave) effect. Sandmo (1977, 1989) was the first to point out this result. The assumption that there is only one risky asset is indeed much more general and much more plausible if the decision is seen as a portfolio choice about how much to invest in the safe asset and how much to invest in the risky market portfolio.
on realized profits, hence, on the difference of final net wealth and the initial capital $m$. The final wealth net of taxes thus amounts to

$$x = a(1 + r(1 - \tau)) + m(\tilde{z} - r)(1 - \tau).$$

(2)

The decision problem of a risk-averse investor maximizing his expected utility amounts to choosing an $m$ so as to maximize

$$Eu(x) = \pi_1 u(x_1) + \pi_2 u(x_2)$$

(3)

with $u$ being a von Neumann-Morgenstern utility function with $u'(x) > 0$ and $u''(x) < 0$, expressing that the investor is non-satiated and risk-averse. We obtain the first order condition for an interior optimum for (3) as

$$E(u'(x)(\tilde{z} - r)) = \pi_1 u'(x_1)(z_1 - r) + \pi_2 u'(x_2)(z_2 - r) = 0$$

(4)

in which the factor $(1 - \tau)$ has been canceled out. While $m$ and $\tau$ do not appear in this condition directly both appear implicitly in (4) as factors that co-determine $x_1$ and $x_2$. Intuitively, equation (4) describes $m(\tau)$ for which the investor cannot further increase his expected utility through marginal variations of $m$. The expected marginal utility from an additional riskily (or safely) invested unit is just zero. Condition (4) is a first order condition. The second order condition, which ensures that solution $m(\tau)$ from equation (4) is in fact a local maximum, is

$$E(u''(x)(\tilde{z} - r)^2) < 0.$$ This sufficient condition is satisfied in the case of risk-averse investors.

What then are the effects of a change in the tax rate? Taking the total differential of (4) with respect to $\tau$ and $m$ leads to

$$\frac{dm}{d\tau} = \frac{m}{(1 - \tau)} + \frac{-\tau a E \left( -\frac{u'(x)}{u'(x)} u'(x)(\tilde{z} - r) \right)}{(1 - \tau) E(u''(x)(\tilde{z} - r)^2)}.$$ (5)

We must now evaluate this expression.

**Case 1: $r = 0$** In case the risk-free interest rate is equal to $r = 0$, (5) changes to

$$\frac{dm}{d\tau} = \frac{m}{(1 - \tau)}.$$ (6)

Term (6) is unambiguously positive. The tax increases the amount the investor uses for the risky investment. This effect of inducing the investor to assume more risk is called the Domar-Musgrave effect and was named after its “inventors,” Evsey Domar and Richard Musgrave (1944). Intuitively, consider an investor who invests everything safely. For this investor $x(0) = a$ is the final wealth. If, instead, the investor chooses some $m_0$ in case of $\tau = 0$, then, for each of the possible outcomes $z_s$ of $\tilde{z}$, this leads to deviations of the final income from $x = a$ by $m_0 z_s$. Now consider a positive tax rate $\tau > 0$. Note that, if the investor follows the rule of (6), the investor chooses $m_\tau = m_0/(1 - \tau)$, this leads to the final wealth $x_i = a + (1 - \tau)m_\tau z_i$. By following (6) the investor can completely undo the effects the tax has on
Figure 1: The portfolio problem for a safe return \( r = 0 \).

the investor’s final wealth and this leaves the investor with exactly the same final wealth as in the state without taxation in each possible state. What is left unclear after this intuitive reasoning is whether or not, after the introduction of a tax, a portfolio that the investor values even higher than any that is attainable in the situation without taxation, is possible. This is not the case. A graphical analysis can illustrate this. Starting with the assumption that \( \tilde{z} \) is a random variable with only two possible values, \( z_1 > r(=0) \) and \( z_2 < r(=0) \), the portfolio problem can be illustrated with a two-state diagram (see Figure 1). On the axis are the final assets of investors \( x_1 \) and \( x_2 \). The expected utility of the investors is as in (3), and the final wealth is

\[
x_s = a + mz_s(1 - r) \text{ for } s = 1, 2.
\]

(7)

In a starting situation without taxes, the investor can, e.g., by choosing \( m = 0 \), realize point \( B \) on the line of risk-free operations, at which \( x_1 = x_2 = a \). By choosing \( m = a \), the investor can move to \( A \), a point with the coordinates \( x_1 = a(1 + z_1) \) and \( x_2 = a(1 + z_2) \). Every point on the connecting line from \( B \) to \( A \) can be realized with an appropriate \( m \in [0, a] \). The set consisting of all these points describes the geometrical position of all the portfolios the investor can realize. Considering his expected utility function (3), the investor chooses exactly that portfolio from \( BA \) which will maximize his expected utility: this portfolio is implicitly determined by the tangential point of an indifference curve with this set of options (we assumed an interior solution). The equality of slopes of the indifference curve and the set of options can formally be expressed as in (4) or

\[
-\frac{\pi_1 u'(x_1)}{\pi_2 u'(x_2)} = \frac{z_2}{z_1},
\]

(8)
where the left-hand side describes the slope of the indifference curve in a point with coordinates \((x_1, x_2)\), which is derived from the total differential of the expected utility function (3), and the right-hand side is equal to the slope of the set of options \(BA\), which is derived from a geometrical consideration concerning the coordinates of the points \(A\) and \(B\). In Figure 1, the optimum is shown as point \(P\). The value of the optimal \(m\) can be seen implicitly on the \(x_1\)-axis. The difference between \(x_1\) associated with \(P\) and the value \(a\) of original assets equals \(m_2\).

Consider now the tax problem. The investor’s preferences concerning the pairs of final assets \((x_1, x_2)\) in the \(x_1 - x_2\) space remains unchanged. However, the tax changes the opportunity set. The investor can still achieve point \(F\) through a risk-free investment. In this case, he has no net revenue and no taxes to pay. If he invests all his assets in the risky asset he cannot reach this point \(A\) any longer. He instead gets to \(F_0\) for a tax rate of \(\tau = 0.25\), for example. He receives \(x_s = a + az_s(1 - 0.25)\) instead of \(x_s = a + az_s\) for \(s = 1, 2\). This point \(A'\) is an element of the original set of options \(BA\), but the distance of \(A'\) to \(B\) is \((1 - \tau)\)-times smaller compared to the distance \(BA\). The same holds true for any other \(m \in [0, a]\). The tax reduces the deviation of the value of the final assets from \(B = (a, a)\) by \((1 - \tau)\); in the graphical example this would be by 75\%. The new set of options is \(BA'\). Which point on this set of options will the investor choose? Point \(P\) is the optimal choice for all points on \(BA\). Hence, \(P\) maximizes the expected utility of all points on a subset \(BA'\) of this original set of options if this subset includes \(P\). The investor cannot reach \(P\) by choosing the same \(m\) as before, but rather through increasing the amount \(m\) that is invested in the risky asset by a factor \(1/(1 - \tau)\).\(^4\)

**Case 2: \(r > 0\)** If the safe interest rate is \(r > 0\), an additional effect of taxation, that is depicted in (5) by the second term, comes into play alongside the effect outlined above. The denominator in the second term in (5) is negative for risk-averse investors. The sign of the numerator is ambiguous in the absence of additional assumptions. More can be said about the effect, however. In preparation for such an interpretation we calculate the effect of an increase in initial assets \(a\). The reaction of the portfolio decision to an increase in the initial assets is

\[
\frac{dm}{da} = \frac{1 + r(1 - \tau)E((-u''(x)x')u'(x)(\bar{z} - r))}{(1 - \tau)E(u''(x)(\bar{z} - r)^2)}. \tag{9}
\]

This “income effect” (9) is very similar to the second term in (5). We can use this similarity and write (5) as

\[
\frac{dm}{d\tau} = \frac{m}{(1 - \tau)} + \frac{-ra}{1 + r(1 - \tau)} \frac{dm}{da}. \tag{10}
\]

\(^4\)Note that this effect does not require expected utility maximization. The effect emerges under a large number of theories of choice under risk and uncertainty. Domar and Musgrave (1944), for instance, used a different theory of choice under uncertainty.
From this, it becomes obvious that, given a positive interest rate, the overall effect of an increase in the tax rate is made up of the sum of two effects: the Domar-Musgrave effect and a term that is proportional to the effect of a reduction of the income derived from taxation.

A further observation is based on John Pratt’s (1964) measures of risk aversion. Pratt defined \((-u''(x)/u'(x))\) as the local measure of absolute risk aversion. It is a measure for the local concavity (“curvature”) of the von Neumann-Morgenstern utility function \(u(x)\). The direction in which this measure changes with the size of the final assets \(x\) determines whether or not the income effect counteracts the Domar-Musgrave effect. If, for example, the Arrow-Pratt measure \((10)\) is constant for all \(x\), then \(-u''(x)/u'(x)\) can be factored out in \((5)\). As a consequence of the first order condition \((4)\), this yields

\[
\frac{dm}{da} = 0
\]

and, therefore,

\[
\frac{dm}{d\tau} = \frac{m}{(1 - \tau)}
\]

holds true also for \(r > 0\). If \(-u''(x)/u'(x)\) is a monotonically increasing function in \(x\), then it can be shown that \(\frac{dm}{da} < 0\). Making use of this in \((10)\) yields

\[
\frac{dm}{d\tau} = \frac{m}{(1 - \tau)} + \frac{-ra}{(1 + r)(1 - \tau)} \frac{dm}{da} > \frac{m}{(1 - \tau)} > 0.
\]

If \(-u''(x)/u'(x)\) is a monotonically decreasing function in \(x\), then it can be shown that \(\frac{dm}{da} > 0\). Accordingly, the income effect may foster or counteract the Domar-Musgrave effect, depending on whether \(u(x)\) shows increasing or decreasing absolute risk aversion.

For the case with \(\tilde{z}\) taking on only two values and \(r > 0\), the portfolio problem can be described as in Figure 2, for a tax rate of \(\tau = 0.25\), with the axis displaying the investor’s final wealth, \(x_1\) and \(x_2\). In the initial scenario without taxes, the investor’s expected utility is determined by equation \((3)\). The final values are now

\[
x_s = a(1 + r) + m(z_s - r) \text{ for } s = 1, 2.
\]

The investor reaches point \(B\), where \(x_1 = x_2 = a(1 + r)\), on the 45-degree line by choosing \(m = 0\). By choosing \(m = a\) the investor can reach \(A\) with the coordinates \(x_1 = a(1 + z_1)\) and \(x_2 = a(1 + z_2)\). Every point on the line connecting \(B\) and \(A\) can be reached by choosing an appropriate \(m \in [0, a]\). The investor chooses point \(P\), the point of tangency, which will result in the highest expected utility value (assuming here an “interior” solution). The equality of slopes of the indifference curve and the set of options can be formally described by

\[
\frac{\pi_1 u'(x_1)}{\pi_2 u'(x_2)} = \frac{-(z_2 - r)}{(z_1 - r)}.
\]
The corresponding \( m \) can be determined on the \( x_1 \)-axis: it is the difference between the \( x_1 \) of point \( P \) and the value \( x_1 = a(1 + \tau) \), which is equal to \( m(z_1 - \tau) \).

The tax with rate \( \tau \) changes the investor’s set of options. When choosing \( m = 0 \) the net portfolio after taxes in Figure 2 is then point \( B' \) with the coordinates

\[
x_1 = x_2 = a(1 + r(1 - \tau)).
\]

The choice \( m = a \) results in point \( A' \) with the coordinates

\[
x_i = a(1 + z_s(1 - \tau)) \quad \text{for} \quad s = 1, 2.
\]

The value of the slope of the line connecting \( B' \) and \( A' \) is then

\[
-\frac{(z_2 - r)(1 - \tau)}{(z_1 - r)(1 - \tau)} = -\frac{-z_2 - r}{z_1 - r},
\]

hence, the same slope as for the original set of options \( BA \). There is a point \( C \) in Figure 2 with the coordinates \((x_1, x_2) = (a, a)\). Geometrical reasoning shows that \( B' \) is a linear projection of the line \( BA \) from point \( C \) with the contraction factor \((1 - \tau)\). Points with equal amounts of investment \( m \) for the situations with and without tax lie on the intersections of straight lines through \( C \) with the sets of options \( B'A' \) and \( BA \). For instance, \( P \) and \( P' \) correspond in the sense that the underlying portfolio leading to these points has the same amount \( m \) of risky investment. The optimum after taxes is a point of tangency of an indifference curve with the set of options \( B'A' \). Without additional assumptions it is unclear whether this point of tangency is to the left or to the right of \( P' \) or even in \( P' \), leading to a lower, higher, or just the same optimal \( m \).

The set of options \( B'A' \) resulting from \( BA \) can also be derived by combining two geometrical operations: the first operation is a contraction of \( BA \) with a fixed point in \( B \), analogous to the contraction discussed in Figure 1 for the case of \( r = 0 \). The second operation is a downward parallel shift of this contracted set of options along \( BC \) toward the origin. If this parallel shift did not occur, the investor’s decision behavior would be identical to the case where \( r = 0 \): He would choose the same portfolio after taxation, hence point \( P \); in order to achieve this, he would have to increase his risky investments by the value of the Domar-Musgrave effect described for the case of \( r = 0 \). Now, however, with the taxation of the return of investment for \( r > 0 \), there is a parallel shift of the set of options toward \( B'A' \). The reaction to this income effect adds to the Domar-Musgrave effect.

**Decreasing returns to scale** We now consider a non-linear set of options. This case has been considered by Mintz (1981). The risky investment may describe an entrepreneurial investment. For the sake of simplicity, suppose here that there is a real investment opportunity for which initial wealth is the only production factor and the entrepreneur needs to choose how much of this wealth to use on this opportunity. The production function, denoted by \( f(m) \), describes the risky output that depends on the capital input. This function is assumed to be twice continuously differentiable with \( f'(m) > 0 \) and \( f''(m) \leq 0 \) for all \( m \in [0, a] \). In
addition we have $f(0) = 0$. The risk for an individual who invested an amount $m$ of his original assets into this activity may, for instance, derive from the riskiness of the selling prices of these produced goods, and we focus again on the case with two possible outcomes. In the good outcome, which occurs with a probability $\pi_1$, the price of the good is $z_1$, in the bad outcome, which occurs with a probability $\pi_2 = 1 - \pi_1$ it is the smaller price $z_2$. With investments of $m$ and without taxation the individual can reach final asset values of

$$x_s = a(1 + r) + z_if(m) - rm$$

in the respective states $s = 1, 2$ of the world.

The slope of the set of options in the point related to $m$ in the two-states-of-the-world diagram is, for the case without taxation,

$$\frac{dx_2}{dx_1} = \frac{dx_2/dm}{dx_1/dm} = \frac{z_2f''(m) - r}{z_1f''(m) - r}.$$  

If we differentiate the expression on the right side of (20) by $m$, the numerator of the derivative is $r(z_1 - z_2)f''(m)$ and has, because of the assumed concavity of the production function, a negative value. The set of options is therefore concave. Because of this, the existence and the uniqueness of the optimal choice of the amount of risky investment $m$ is secured.

We will now examine the tax. We look at the tax effects for the case of a generic linear income tax with tax rate $\tau$ and we allow for an exemption $b \geq 0$. The two values of possible final wealth are

$$x_s = a(1 + r(1 - \tau)) + b\tau + (1 - \tau)(z_sf(m) - rm) \text{ for } s = 1, 2.$$
In a two-state diagram, a linear tax leads to a projection of the original set of options with the projection center \((a + b, a + b)\) and the projection factor \(1 - \tau\). A synthetic income tax with \(b = 0\) is presented in Figure 3. In it, a situation is described in which \(z_2 f'(0) < r\) applies, i.e., the slope of the set of options is decreasing everywhere. However, this assumption is not crucial for the effect of the tax.

The analysis of the tax effects is based on the fact that, for every given value \(m \in [0, a]\) in a linear tax model, the slope of the set of options stays the same before and after taxation. This is the case because the slope of the set of options after tax, from (21), can be calculated as

\[
\frac{dx_2}{dx_1} = \frac{dx_2/dm}{dx_1/dm} = \frac{(1 - \tau)(z_2 f'(m) - r)}{(1 - \tau)(z_1 f'(m) - r)}.
\] (22)

From this, a general criterion concerning how linear taxation can change the level of investment in a risky project can be derived: For this let \(P\) and \(P'\) be points that correspond to the same amount of investment, and let this be the optimal amount of investment in the case of \(\tau = 0\). Now, an increase in \(m\) results if the indifference curve passing through point \(P'\) is steeper than the indifference curve passing through \(P\). Such a situation is depicted in Figure 3. This criterion is always fulfilled if the condition

\[
a(1 + r(1 - \tau)) + \tau b + (1 - \tau)(z_2 f(m) - rm) > a + b
\] (23)

holds true for the originally optimal investment amount of \(m\), i.e., if a loss is suffered in the case of unfavorable final assets \((a + b)\). In particular, if we allow for the deductibility of the safe returns of investment, i.e., \(b = ar\), we arrive at a situation in which the original Domar-Musgrave effect emerges.

**Imperfect Loss Offset** So far we have assumed that the tax is strictly proportional to the investor’s revenues. The revenue in state \(s\) (meaning if \(z\) assumes the value \(z_s\)) results as the difference between the gross value of final assets \(a(1 + r) + m(z_s - r)\) and the value of initial wealth \(a\). If the difference is positive, the investor has to pay a proportional share in taxes. If the difference is negative, the investor may receive a payment from the state. In practice, even though such direct subsidies occur only rarely, this is not a bad assumption: even if the government does not fully participate in losses from an investment in this literal format, there are provisions which have similar economic effects. Tax law in many countries allows firms to offset losses from one period with taxable profits from previous periods, if there are no available profits from the current period from other activities.⁵ The firm then gets some of its previously paid taxes back in the event of a loss. From an economic point of view, this is equivalent to a government’s participation in the current losses. If loss-carry-back provisions fall short of a firm’s losses, there is the possibility of a loss-carry-forward and the option to offset them with future profits. Tax-loss carry-forward is not completely equivalent

⁵The number of years for which a loss-carry-back is allowed as well as the total size of loss-carry-backs may typically be limited.
The tax advantage of a tax-loss carry-back therefore has a higher present value than the tax advantage of a tax-loss carry-forward. In practice these limitations lead to an incomplete loss offset, and we now discuss its consequences.

Our analysis here is for the easiest case: $z_1 > \tau = 0 > z_2$ and a graphical representation is given in Figure 4. Let the investor’s choice of risky investment for $\tau = 0$ be $m$ such that the optimal value of $m$ fulfills the first order condition

$$\pi_1 u'(a + m z_1)z_1 + \pi_2 u'(a + m z_2)z_2 = 0. \quad (24)$$

This optimal value is generally a function of the tax $\tau$ which we consider, and we denote the optimal $m(\tau)$ for $\tau = 0$ as $m(0)$. In the absence of loss-offset provisions, the investor pays a proportional tax on the risky returns, $\tau m(0)z_1$, but bears the full gross burden of losses $m(0)z_2$. Accordingly, for a positive tax $\tau > 0$, the investor has an incentive to choose a higher $m$ than $m(0)$ if

$$\pi_1 u'(a + m)(1 - \tau)z_1(1 - \tau)z_1 + \pi_2 u'(a + m z_2)z_2 > 0 \quad (25)$$

at $m = m(0)$, i.e., if, at $m(0)$, the expected utility of a further extension of the riskily invested value is positive. A comparison of (24) and (25) shows that this case applies if

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\[ u'(a + m(1 - \tau)z_1)(1 - \tau)z_1 > u'(a + mz_1)z_1 \]  

(26)

holds at \( m = m(0) \). For this condition, it is sufficient that \( u'(a + z)z \), as a function of \( z \), is monotonically declining in the interval of \([(1 - \tau)z_1, z_1]\), i.e., if \( u''(a + z)z + u'(a + z) < 0 \). Defining \( y := a + z \), this condition can be written as

\[ -\frac{u''(y)y(y - a)}{u'(y)} > 1 \]  

(27)

for all \( y \in [a + (1 - \tau)z_1, a + z_1] \).

Condition (27) implies that only risk-averse investors whose index of relative risk aversion \( -\frac{u''(y)y(y - a)}{u'(y)} \) is high in the relevant interval will increase their risky investment when in the case without loss offset the tax rate is increased. The impact of a high degree of relative risk aversion\(^7\) is, however, mitigated by the second term \( (y - a)/y = z/(a + z) < 1 \) on the left-hand side of (27). If the index of relative aversion is constant or decreasing in wealth then – for a given risky project – condition (27) will be violated if initial wealth is sufficiently high. Hence, rich investors will react with a reduction of their risky investment when the tax rate rises and loss-offset provisions are absent.

Incomplete loss compensation matters only for risk-taking choices \( m \) which lead to potentially negative returns. For the general case with \( r > 0 \), there is some amount of taxable income that accrues on the amount of wealth that is safely invested, and some of the taxable profits can be consolidated with the losses to reduce taxable income. However, the losses may exceed this component of income in the loss state, in which case the incomplete loss offset becomes relevant. The effect of the tax then crucially depends on the position of the investor on the set of options before and after taxation. A distinction of cases for risk neutrality can be found in Eeckhoudt et al. (1997).

The changes in the attractiveness of risky investments could be avoided if the legislators were to create or improve the possibilities for investors to trade tax deductible losses among them. This sort of trade is technically feasible. Companies, for instance, which carry major tax losses may merge with highly profitable companies, and the taxable profits may be consolidated with these losses. However, tax law has become increasingly restrictive, one reason being that companies use taxable losses to shift and consolidate these with gains across jurisdictions, making use of differences in the effective tax rates in these countries. Also the attempt to increase the overall tax revenue may be one of the motivations for this trend. One should be aware, however, that this practice introduces an asymmetry in the tax code and may negatively affect the efficiency of the allocation of risks. In particular, it creates an asymmetric treatment between different types of firms. Large and well-diversified companies with deep pockets can consolidate tax losses and taxable profits in different parts of the

\(^7\) A recent empirical study by Chiappori and Paiella (2011) shows a median value for the index of relative risk aversion slightly below 2 but with much heterogeneity among individuals. For an overview of former empirical studies on relative risk aversion see Meyer and Meyer (2005).
enterprise and over time. Start-ups which have not only a large upside potential but also a considerable downside risk, are treated worse: if they are successful, they pay the full amount of the tax, but in the event of failure, bankruptcy looms. The losses cannot be consolidated with future profits in this case, and also the scope for consolidation across different branches inside the company are typically non-existent.

**Limited Liability** A further important asymmetry as regards the tax treatment of losses and gains emerges if capital owners make their investments under conditions of limited liability. These are situations in which the investor does not bear the full losses in loss states, but can shift some of these losses to other players. The problem of overinvestment in risky activities if decision makers do not directly bear the increased downside risk if they increase their risk taking is well known and has been thoroughly studied by Sinn (1982). We do not endogenize the reasons for limited liability here. Note also that limited liability and the investment incentives it creates are anticipated by creditors and other business partners. Limited liability may come along with the legal form of company chosen. For private investors it may be based on a social consensus on guaranteed minimum incomes in a society or by the simple fact that the losses incurred exceed the total wealth of a person. And for some companies the limited liability may be the outcome of implicit guarantees given to such companies because of their special role in the functioning of the private economy – an issue that is particularly relevant for large financial institutions and that has attracted much attention in the context of major financial crises. Also, we analyze this problem in the absence of co-funding of investment projects by debt – an aspect that is important but complicates

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8 See, for instance, Hellwig (1981) for an analysis of limited liability and the validity of the Modigliani-Miller theorem.
the analysis considerably.

We extend the simple partial analytic investment problem of the previous sections with an investor who is in command of an initial wealth of $a$ and can invest an amount $(a - m)$ safely with a zero interest rate $(r = 0)$.

The remainder is invested in a risky project that yields $z_1 > 0$ in the good state and $z_2 < 0$ in the bad state, where we limit $m$ to $m \in [0, \tilde{m}]$. We further assume that the investor has an exogenously given reservation income which is reached should his income after investment and taxation fall below this reservation income, and we set this reservation income equal to zero. In this case the income of the investor is

$$x_1(m) = a + mz_1(1 - \tau)$$

in the good state and

$$x_2(m) = \max\{a + mz_2(1 - \tau), 0\}$$

in the bad state. The investor maximizes the expected utility $Eu(m) = \pi_1u(x_1) + \pi_2u(x_2)$.

The locus of investment opportunities and the combinations of $(x_1(m, \tau), x_2(m, \tau))$ is depicted in Figure 5. The curve $BA$ corresponds to the curve $BA$ in Figure 1, with $B$ the point reached for $m = 0$ and $A$ reached by $m = a$. However, limited liability implies that the budget curve is kinked at the point where $x_2(m)$ would fall below zero and becomes horizontal. For sufficiently high $m$ the wealth in the bad state falls below 0, but limited liability implies that the investor does not end up with $a + mz_2$, but with $x_2 = 0$.

In the absence of taxation, there are two possible points that may maximize expected utility. One is a point of tangency of an indifference curve with the opportunity set along the segment $BD$ similar to point $P$ in Figure 1. The other possible maximum is at $C$. As drawn in the figure, the maximum is reached at $C$.

Starting from this situation we can discuss the role of a proportional tax on the returns $mz_2$. Such a proportional tax changes the opportunity set that applies in the absence of limited liability precisely as described in the context of Figure 1. For a tax rate $\tau$ the opportunity set $BA$ has a fixed point at $B$, but $BA$ contracts into $BA'$, where the distance $BA'$ is just $(1 - \tau)$ times the distance $BA$. Taking limited liability into consideration, the actual opportunity set also changes, from $BDC$ to $BDC'$, where $C'$ is located along the former segment $BC$ vertically above $a + \tilde{m}z_1(1 - \tau)$. A sufficiently large tax reduces the horizontal segment of $BDC$ sufficiently to induce an optimal choice of $m$ along the downward-sloping segment of the opportunity set. Accordingly, if the tax is sufficiently large, it changes the investment equilibrium from $m = \tilde{m}$ to an amount that corresponds to the amount determined in the absence of limited liability. This is the point $P$ that has been derived in Section 2.1. Once the switch from $m = \tilde{m}$ to this interior $m$ takes place, further increases in $\tau$ will have effects similar to those discussed in Section 2.1.

At this point it is important to emphasize the partial nature of these results. In particular, the analysis is silent about the issue of who bears the losses $a - mz_2$, i.e., who

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9 For a parallel and more elaborate approach with a different investment technology, see Ewert and Niemann (2012).
pays for the loss which is generated by the investor’s high-risk exposure if things go wrong. In what follows we turn to general equilibrium effects. We first reconsider the effects studied in Section 2.1 in a general equilibrium framework and then turn to issues such as limited liability. The section shows that general equilibrium considerations fundamentally change the picture.

3 General Equilibrium

So far we have concentrated on the partial analysis of a single investor, not taking into account that taxation and the use of these revenues have important feedback effects, and that the tax revenue stays inside the economy. These effects have been highlighted by Bulow and Summers (1984) in the context of a capital asset pricing model approach, and by Konrad (1991) within the context of an Arrow-Debreu general equilibrium model.

Here we broaden the analysis by taking into account what happens with the tax proceeds as in Buchholz and Konrad (2000). Let there be a number $n$ of investors indexed by $i$ or $j$, with $n$ not necessarily being large. Let them all be endowed with initial capital $a$ and let their investment choice be characterized by an investment of $a - m^j$ in the safe asset with return $r$, and an investment $m^j$ in the risky market portfolio, the latter being seen as the sum of all risky assets in the economy. Let the risk characteristics of this market portfolio be described by $\tilde{\zeta}$ as defined in Section 2.1., for the case with only two states of nature, denoted
by $s = 1, 2$. The aggregate tax revenue is

$$T_s = \sum_{j=1}^{n} (\tau a r + \tau m^j (z_s - r))$$

(28)

for $s = 1, 2$. Let the tax revenue that is returned to individual $j$ in state $s$ be denoted by

$$L_{js} = L_s = T_s/n$$

(29)

for $s = 1, 2$. We show that the asset choice $m$ which is a symmetric Nash equilibrium of this situation in the absence of taxes will also be a Nash equilibrium even after the introduction of the tax transfer system under standard Nash conjectures. The final wealth of investor $j$ in the states $s = 1, 2$ can then be described as

$$x_s^j = a(1 + r(1 - \tau)) + m^j (z_s - r)(1 - \tau) + L_{js}.$$  

(30)

If all individuals choose the same $m$, this expression is reduced to

$$x_s^j = a(1 + r) + m(z_s - r).$$

(31)

This shows that feasible symmetric allocations $(x_1, x_2)$ are determined by the same equation as in the absence of taxation, and the opportunity set for the aggregate economy remains the same for the economy.

Consider the allocation in which all individuals invest the same share $m$ of their initial assets riskily in the situations with and without a tax. Note that the final wealth that emerges for each individual is the same for $\tau = 0$ and for a positive $\tau$. This does not imply that the choice of the same $m$ also describes the symmetric Nash equilibrium that results after the introduction of the tax. In order to show that it does, we have to prove that it is the optimal decision for an investor $i$ to choose precisely the same $m^i = m$ as in the equilibrium for $\tau = 0$, if he believes that all other investors $j$ choose this very $m$. Investor $i$, hence has to solve the problem of maximizing the following expected utility by the choice of $m^i$:

$$\sum_{s=1}^{2} \pi_s u(a(1 + r) + \frac{n-1}{n} \tau m(z_s - r) + m^i(z_s - r)(1 - \tau \frac{n-1}{n})).$$

(32)

Let us denote the investor $i$’s realized final wealth values after the solution of the optimization problem as $\hat{x}_1^i$ and $\hat{x}_2^i$. Then the marginal condition for the maximization of the investor’s expected utility is:

$$\pi_1 u'(\hat{x}_1^i)(z_1 - r)(1 - \tau \frac{n-1}{n}) + \pi_2 u'(\hat{x}_2^i)(z_2 - r)(1 - \tau \frac{n-1}{n}) = 0.$$  

(33)

This marginal condition for the optimal solution after taxation is equivalent to the case without taxes, if $\hat{x}_1^i = x_1^i$. But the latter is the case if $m^j = m$ for all $j$. This is a strong super-neutrality result. The tax does not have any behavioral effects for the taxpayers. It
does not even have income effects. And this neutrality result is not limited to large economies with a large number of tax payers. It also applies if the set of taxpayers is finite.

The combination of a proportional income tax and an even distribution of the reimbursement of the profits from taxes for all investors leaves the initial portfolio allocation unchanged and, hence, allocation neutral. Both the Domar-Musgrave effect and the income effect are canceled out by the transfer system. This shows that the neutrality result is independent of the number of investors \( n \) and, hence, already applies in the case of \( n = 2 \). The analysis suggests that linear taxes do not have any effect on the risk allocation in a world with perfect and complete capital markets. Such markets consolidate all idiosyncratic risks. The aggregate risk that remains if such markets exist cannot be eliminated through a tax, as all diversification that is feasible has already taken place in these markets. For this reason the government cannot achieve further risk sharing in this case.

The formal analysis above made an important symmetry assumption. In a model with strict symmetry the tax on risky returns has two mutually offsetting effects. The amount of risk remaining with the investor after taxes is reduced by the tax. On the other hand, the investor receives risky lump-sum payments. These two amounts of risk are just equal. The lump-sum transfer really is a perfect substitute for the reduction in voluntary acquisition of risk in the capital market. Asymmetry along one or several dimensions can change the result. If tax payments and lump-sum transfers do not balance, some individuals may experience a positive income effect and others may experience a negative income effect. Income effects lead to changes in the desired amount of risk-taking. These changes do not necessarily balance out in the aggregate. Hence, the aggregate amount of risk-taking can change. If, for example, income is redistributed from individual 1 to individual 2 because of a lump-sum tax, it is not certain that the now poorer individual 1 will reduce the amount of risky investments by exactly that amount by which 2 expands his risky investments.

Asymmetries need not destroy neutrality. For instance, Konrad (1991) treats the case of heterogeneous preferences and heterogeneous endowments. He allows that the government makes lump-sum redistributions of proportional shares in the total tax revenue. He treats the production of the risky assets as endogenous. He analyzes the Walrasian equilibrium and shows that a proportional tax on risky returns is fully neutral for \( r = 0 \). The tax does not affect asset prices and does not affect the portfolio choices of investors nor their net payoffs in the equilibrium.

Intuitively, in the Walrasian equilibrium for \( r = 0 \) the market price for an additional share in the market portfolio that constitutes the 'risky asset' is equal to the price of the safe asset, even if the expected return of the risky asset is positive. Individuals are just indifferent between whether to keep the last unit of their wealth in the form of a safe asset with zero net return or whether to purchase one further unit of the risky asset which has a net return that has a positive expected value, but is risky. They attribute a willingness-to-pay of zero for the purchase of an additional unit of this net-return, that is, for \( \tilde{\zeta} \). By way of a proportional tax, the government assumes a share in this risk, that is, a risky return equal to \( \tau \tilde{\zeta} \), and the government redistributes this revenue in equal proportions to the individuals. But as
the equilibrium market price of the vector of state-contingent payouts has a market price of zero, this also holds for the market value of the tax revenues as a whole and for the share in these revenues that goes to the various taxpayers. Accordingly, the taxpayers attribute a net value of zero to both the tax payments and the lump-sum transfers. However, the tax and the transfer changes the composition of their portfolio allocation and they re-establish the original (individually optimal) structure by their market transactions. Furthermore, this implies that the total sum of portfolios, which individuals desire to hold for unchanged prices, remains unchanged.

**Limited liability** It is also interesting to see how the limited liability of investors changes the results in the general equilibrium analysis, and how the results compare to the partial equilibrium analysis on limited liability and taxation. In general, if a decision maker has limited liability, this means that the decision maker may have a loss that exceeds the value of assets available to cover this loss. If such a loss occurs, someone must cover the difference between the loss and the amount that is covered by the decision maker. We assume here that this difference is covered by public subsidies. In turn, this implies that these subsidies must be financed by general taxes.

Several examples come to mind that fit this assumption. An economy with systemic banks is an example. Each bank has some equity capital and additional funds (e.g., deposits) available for investment. If the investment turns bad and a loss occurs that exceeds the equity capital, the government bails out the bank because the bank is systemic ("too big to fail"). It covers the loss that exceeds the equity capital. Deposits are basically unaffected. But the funds used for the bailout need to be raised by general taxes. The bank itself is owned by shareholders. These shareholders of the bank have an initial wealth. They provide the amount of equity that is needed to be permitted to operate the bank, and this equity capital is lower than their total wealth. In turn, in an economy with an homogeneous set of individuals, these shareholders also constitute the general public which has to pay the tax that is needed for the bailout.

More formally, to imbed the portfolio problem with limited liability from Section 2 into a general equilibrium framework we keep up the assumption that there is a perfect and complete capital market, such that \( \tilde{\xi} \) is the risky payoff of the market portfolio. Each private individual is in command of an amount of initial wealth of size \( \alpha \). We disentangle the wealth of the individuals from the wealth that is directly liable if the investment fails. Suppose each individual can set up a firm. This firm requires an injection of some equity capital that is equal to \( k < a \). The firm has limited liability, meaning that losses exceeding this capital \( k \) are covered not by the firm, but – in our framework – by the government. The firm can invest and purchase risky assets for an amount \( m \in [0, \bar{m}] \). These assets are shares in the market portfolio of risky assets. The return of these assets is \( mz_1 > 0 \) in the good state and \( mz_2 < 0 \) in the bad state. Note that a sufficiently high investment \( m > -k/z_2 \) may cause a loss for

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10 A similar claim is made by Bulow and Summers (1984, p. 24) who argue that "The government is not distorting investment because it is extracting a claim of zero market value..."
the firm that exceeds its equity capital. Limited liability implies that the owner of the firm is not liable if the firm’s losses of the firm exceed the firm’s equity capital. Someone has to cover the losses exceeding $k$ in this case, and we assume that the government ‘bails out’ the company in this case and finances such bail-out payments from the general taxes.

We denote the net value of the firm of investor $i$ by $\theta^i$, which is

$$\theta^i_s = \max[k + (1 - \tau)m^i z_s, 0].$$

It is equal to the equity capital plus the returns of investment, net of taxation if this sum is positive, and it is equal to zero if this sum is negative. Individuals’ final wealth consists of

$$x^i_s = (a - k) + \theta^i_s + \frac{1}{n} \sum_{i=1}^{n} \tau m^i z_s + \frac{1}{n} T_s.$$

This final wealth consists of several terms. The first term is initial wealth minus the equity capital. The second term is the net value of the firm as defined in (34). The third term takes into consideration that the government taxes the risky returns and redistributes these symmetrically among the $n$ individuals in the private sector. Finally, $T_s$ is non-positive and is the amount of government bailout payments needed to cover the firms’ losses. Hence, in the good state, $T_1 = 0$. In the bad state, whether the government has to pay to cover losses depends on the investment policy of firms, and the amount of payments is

$$T_2 = \sum_{i=1}^{n} \min\{k + (1 - \tau)m^i z_2, 0\}.$$ 

Each individual firm owner chooses the $m^i$ for his firm. If all $m^i$ are chosen, the state of nature is revealed and payoffs are determined.

In order to make sure that the total wealth in the economy remains non-negative, we need to make an assumption about the relationship between total wealth and the maximum aggregate loss that may occur. We assume that

$$\bar{m} z_2 = -a.$$ 

This condition makes sure that the total wealth in the economy is sufficient to cover the maximum loss that can emerge if all firms choose maximum risk-taking and if the bad state of the world emerges.

An equilibrium is characterized by a vector of investment choices $(m^1, ..., m^n)$ such that $m^i$ is an optimal reply for the investment choices of all other individuals, for all $i = 1, ..., n$. There are two candidates for an equilibrium. One candidate is $m^i = \bar{m}$. The other equilibrium candidate is $m^i = m(\tau)$, where $m(\tau)$ is defined by (4) for $r = 0$. To confirm the equilibrium with excessive risk taking in particular, we use a heuristic and graphical approach. For this purpose we construct the graphical representation of players’ choice sets. Consider first the equilibrium candidate with $m^i = \bar{m}$ for all $i = 1, ..., n$. Figure 6 shows the individual budget set of a player $i$ for a given $\tau$ that emerges if all players $j \neq i$ choose $m^j = \bar{m}$, for different
choices of $m^i$. This budget set starts at point $B'$ which is already distant from the 45-degree line and has the coordinates

$$(x_1^i, x_2^i) = (a + \frac{n-1}{n} \tau \bar{m} z_1; a + \frac{n-1}{n} (\bar{m} z_2 + k)),$$

for a range of $\tau$ with $k + (1 - \tau)m^i z_2 < 0$.

To verify coordinate $x_1^i$, note that the player chooses $m^i = 0$, but receives a positive net transfer equal to $1/n$ of the aggregate tax revenue, whereas all lump-sum taxes and lump-sum transfers net out for each individual. To verify coordinate $x_2^i$, note first that each of the $n-1$ firms generates a gross loss equal to $\bar{m} z_2 = a$ in the bad state. After taxation, this loss is $(1 - \tau)\bar{m} z_2$. The firm can cover the share $k$ of this loss, but the loss $(1 - \tau)\bar{m} z_2 + k$ needs to be covered by the government. In order to cover this loss, the lump-sum tax must be equal to

$$b = \frac{n-1}{n}(-(1 - \tau)\bar{m} z_2 - k).$$

(39)

Also, the government redistributes the tax losses in this state, and, on a per-capita basis, these are

$$\frac{n-1}{n} \tau \bar{m} z_2.$$

(40)

The slope of the linear segment starting in $B'$ is $z_2/z_1$: a marginal unit $dm^i$ generates additional income $(1 - \frac{n-1}{n} \tau) z_2$ in the respective state $s$. This is because for small $m^i$ the losses are covered by the equity capital $k$ of $i$’s firm. For an even higher $m^i$ there is a point at which $k$ no longer covers all losses $m^i z_2 (1 - \tau)$ emerging inside the firm. The budget constraint reaches a kink. This is the case if $k = -(1 - \tau)m^i z_2$. At this point, for an additional marginal
unit of \( m^i \), the final wealth \( x_1 \) continues to increase by \((1 - \frac{n - 1}{n} \tau)z_1 \). The final wealth \( x_2 \), however, follows a different pattern. It consists of several components. First, as the firm reaches its limited liability, \( \theta_2 \) remains unchanged at 0. However, \( i \) has to contribute to cover part of the additional losses \( z_2(1 - \tau) \) by \( \frac{\partial x_2}{\partial \mu} = -\frac{1}{n}(1 - \tau)z_2 \). Also, \( i \) receives an additional share in the tax losses, which is \(-\frac{1}{n}\tau z_2 \). In total, \( i \) bears \( \frac{1}{n} \) of the additional losses in state 2, and \( x_2 \) reduces by \(-\frac{1}{n}z_2 \). Accordingly, the slope of the budget line at the kink is

\[
\frac{dx_2}{dx_1} = \frac{1}{(1 - \frac{n - 1}{n} \tau)z_1}
\]

until \( m^i = \bar{m} \) is reached. This flatter line segment ends for \( m^i = \bar{m} \) with a coordinate that has

\[
(x^i_1, x^i_2) = (a + \bar{m}z_1; a + \bar{m}z_2).
\]

The outcome with \( m^i = \bar{m} \) is an equilibrium if the point \( A \) yields \( i \) the highest expected utility. This confirms that \( m^i = \bar{m} \) can emerge as an equilibrium and this equilibrium has excessive risk-taking. Note that the segment \( B'C \) becomes more and more attractive if \( \tau \) increases. Figure 6 shows the situation for \( \tau = 1/2 \) and \( n = 2 \). A sufficiently high \( \tau \) destroys the equilibrium with excessive risk-taking. For instance, if

\[
k - (1 - \tau)\bar{m}z_2 > 0,
\]

then limited liability never becomes binding. The firm may make losses after tax, but these are always smaller than the liable amount of equity \( k \) which belongs to the firm. Accordingly, for sufficiently high taxes the outcome is equivalent to the case without limited liability, leading to an equilibrium with \( m^i = m(0) \) as in (4) for all \( i = 1, \ldots, n \).

4 Incomplete Information

Strong neutrality results apply for perfect and complete capital markets. However, capital markets are notoriously incomplete and imperfect. Can the government, by way of taxation, consolidate risk and generate a surplus, similar to the surplus that would otherwise be created by capital markets or by a functioning insurance market? One may argue that a tax on risky returns may consolidate risk, and this may be beneficial for society. As was discussed in the introduction, some researchers have argued along these lines. However, the fact that capital markets are incomplete and seemingly offer easy gains from risk consolidation itself, and that private equity markets do not pick these low-hanging fruits should make us suspicious. Incomplete risk consolidation may be due to good reasons. If, for example, employees could fully insure their income and career risks, there would certainly be an enormous disincentive effect.

The principal-agent theory tells us that there exists a tradeoff between risk diversification and incentives. Players often deliberately assume idiosyncratic risks in private markets, because this risk-taking provides them with adequate incentives. The direct participation
of managers in a company’s success and in its idiosyncratic risks incentivizes them to work for the company’s success. The government could achieve risk consolidation through a 100-percent tax on the salaries of managers and redistribute these taxes in a complete lump sum fashion. But this would not be desirable. A meaningful study of the role of a tax on risky returns in the absence of complete capital markets requires that this study treats the amount of risk diversification in private markets as endogenous. It must take into account the reasons why the private markets (often voluntarily) abstain from the full diversification of idiosyncratic risk. The risk consolidating effect of the national tax revenue may have counterproductive incentive effects in the private economy – incentive effects that were the reasons why the private markets abstained from this risk consolidation. In this chapter, we will analyze more closely informational reasons for incomplete risk diversification in private capital markets and the effect of taxes in these cases.

4.1 Moral Hazard

First, we study the effect of a capital income tax on the risk allocation with possible moral hazard by entrepreneurs. We consider entrepreneurs who found and manage a firm in an economy with perfectly well-functioning capital markets by following the exposition in Buchholz and Konrad (2000). Each entrepreneur has an initial endowment of $a$ and can operate an entrepreneurial project: his firm. The project’s revenue $\tilde{\zeta}$ depends on the state of the world $(s = 1, 2)$ as regards this firm’s profits, and these amount to either $z_1$ or $z_2$ with $z_1 > z_2$. The probability $\pi_1(e)$ for the good state to emerge is now a function of $e$, the entrepreneurial effort for the company. The payoff $z_2$ results with the remaining probability $\pi_2(e) = 1 - \pi_1(e)$. The entrepreneur can choose between a high effort $e = \bar{e}$ and a low effort $e = 0$. A high working effort leads to a higher probability of success; in effect we have $\pi_1(\bar{e}) > \pi_1(0)$. We do not need to index them as we can focus on one representative firm from this group.

The entrepreneur, who founded and now manages this firm maximizes

$$\pi_1(e)u(x_1) + (1 - \pi_1(e))u(x_2) - e. \quad (41)$$

Here, $x_s$ is the value of his final income or wealth for the states of the world $s = 1, 2$, and effort $e \in \{0, \bar{e}\}$ generates disutility of exactly this amount.

The entrepreneur could just own and operate the firm himself. But doing this would not be optimal. Given that the profit risk in his firm typically is stochastically independent from profit risks in other firms, there is a potential gain from risk diversification. More precisely, if there is a number of such firms, and if all the risky profits of these firms are stochastically independent, then we could add up all these profits, divide the sum by the number of firms, and receive an amount of profit that, by the law of large numbers, would be an almost safe amount and equal to the expected profit of one of these firms. Assuming that a capital market provides opportunities for such risk sharing the single entrepreneur could sell a fraction of the stochastic profits of his firm for the equilibrium market price. We denote the fraction of company shares he sells by $(1 - q)$. The stock market value of the
The whole company is denoted $p$. This implies that the entrepreneur’s final wealth is

$$x_s = a + q z_s + (1 - q)p \text{ for } s = 1, 2.$$ \hfill (42)

Several considerations govern the decision on $(1 - q)$ and the equilibrium value of $p$, the most important one being that there will be a relationship between the entrepreneur’s incentives to provide effort and the share in $z$ sold in the capital market. This incentive feeds back on the price paid for shares in this random profit. We start analyzing this problem with a case in which capital market investors can observe the true effort $e$ chosen by the entrepreneur. This case of perfect observability serves as a benchmark for the analysis of what happens if $e$ is not observable for capital market investors.

**The complete information benchmark** Suppose first that the capital market investors could observe or enforce the entrepreneur’s effort prior to buying shares in the profit of this firm. In this case the sales contract of the firm’s shares would typically also determine the entrepreneur’s effort. Under the assumption of perfect competition amongst the individual capital market investors and in view of the stochastic independence of the risks of all such single projects, the entrepreneur would receive an offer for the company share which is exactly equal to the expected revenue of the respective project. Under the choice of $e \in \{0, \bar{e}\}$, the entrepreneur would obtain the price

$$p_e = \pi_1(e) z_1 + \pi_2(e) z_2.$$ \hfill (43)

To make maximum use of the scope for risk diversification he would sell his entire company at this price. The capital market investors would be indifferent about whether or not to buy shares in this firm at this price, so this price determines the equilibrium price and this price is a function of the entrepreneurial effort. If

$$u(a + p_e) - \bar{e} > u(a + p_0)$$ \hfill (44)

we would reach the solution that all entrepreneurs would choose the high effort level $e = \bar{e}$ and the price of the company shares would be $p_e$.

**Unobservable effort** We now turn to the case with unobservable effort. In this case the contract by which the entrepreneur sells a share in his company cannot be conditional on the entrepreneur’s actually chosen effort level. As the choice of effort is made once the share $(1 - q)$ is determined, the entrepreneur will choose the effort level that maximizes the entrepreneur’s payoff. The determination of equilibrium becomes less straightforward than in the benchmark case here. Borrowing from the insurance theory literature on moral hazard, we concentrate on a moral-hazard equilibrium in which only a fraction of the firm is sold in the capital market.\footnote{A second, less interesting equilibrium has $e = 0$. In this equilibrium the entirety of the enterprise is sold in the capital market at price $p_0$. The entrepreneur thus achieves a state of perfect income security. The payoff} To arrive at this equilibrium, we assume the following institutional...
framework, players’ actions and their timing. First all firms are established – a step that we consider to be exogenous and not a matter of choice. Next, entrepreneurs have to choose whether to go public. There are portfolio investors who are willing to purchase shares in the firm at a price that equals the expected profit associated with these shares. Entrepreneurs now make a going-public offer: they offer a fraction \((1 - q)\) of the profits of their firm, based on a market value of the firm of \(p\), and they firmly commit not to sell the remaining part \(q\) in this firm later, but to retain this part in their own portfolio. The investors decide whether or not to purchase at this price. Once the going-public process is completed, the entrepreneur chooses the \(e\) that suits him best, then firm profits materialize and are distributed according to the share ownership.

Solving this game backward, we first consider the entrepreneur’s choice of \(e\) for a given share ownership that is characterized by \(q\). We find that the entrepreneur will choose 
\[ e = \bar{e} \] 
if
\[ \pi_1(\bar{e})u(x_1) + (1 - \pi_1(\bar{e}))u(x_2) - \bar{e} \geq \pi_1(0)u(x_1) + (1 - \pi_1(0))u(x_2) \] 
(45)
with
\[ x_s(q) = a + qz_s + (1 - q)p_\ell. \]
The entrepreneur will choose \(e = 0\) otherwise. We call this condition the incentive compatibility condition. This condition needs to hold to provide the entrepreneur with appropriate incentives to expend high effort. The condition can also be written as
\[ u(x_1) - u(x_2) \geq \frac{\bar{e}}{\pi_1(\bar{e}) - \pi_1(0)}. \] 
(46)
Note that the right-hand side of (46) is independent of \(q\). The left-hand side is monotonically increasing in \(q\). This holds because
\[ \frac{d}{dq}(u(x_1) - u(x_2)) = u'(x_1)(z_1 - p_\ell) - u'(x_2)(z_2 - p_\ell) \]
and \((z_1 - p_\ell) > 0\) and \((z_2 - p_\ell) < 0\). Hence, condition (46) determines a critical \(q^*\). The entrepreneur can be expected to expend high effort only if the entrepreneur’s retained shares in the firm are equal or higher than this \(q^*\).

Moving on to the ‘going public’ stage, we can consider what the most attractive contract \((q, p)\) is from the perspective of the entrepreneur by what he can offer and what is accepted by the portfolio investors. These investors observe \(q\). Hence, they attribute a value to the profits of the firm that is equal to \(p_\ell\) if the contract entails a retained share that is at least as large as this critical \(q\), and a value of \(p_0\) otherwise. When the entrepreneur decides on \((q, p)\) he takes this constraint on \(p(q)\) into consideration. Only two candidates for an optimal contract \((q, p)\) emerge: these are \((0, p_0)\) and \((q^*, p_\ell)\). Here and in what follows we consider a situation in which this optimum is \((q^*, p_\ell)\). For this to be the case the expected value added \((\pi(\bar{e}) - \pi(0))(z_1 - z_2)\) through the entrepreneur’s choice of a high effort level needs to be comparatively high in proportion to the utility reduction resulting from the entrepreneur’s high effort.

of the entrepreneur is lower in this outcome than in the benchmark situation with complete information by (44).
The role of taxes  

Now we introduce a proportional profit tax with a tax rate $\tau$. We continue to assume that a complete consolidation of risks can take place when the profit risk of this firm is consolidated with the profit risks of other firms inside the market portfolio. Note that the same consolidation of risk emerges within the tax revenues of such a tax and its redistribution, as there are many firms with stochastically independent risks paying this tax. Further, we assume that the tax revenues are redistributed among the entrepreneurs, and every entrepreneur receives a safe transfer from the redistribution of the tax revenues for all firms, with the amount redistributed of

$$S_e = \tau p_e$$

such that the state-dependent final wealth of the entrepreneur amounts to

$$x_s(q) = a + q(1 - \tau)z_s + (1 - q)(1 - \tau)p_e + S_e$$

for $e \in \{0, \bar{e}\}$ for a given choice of $q$. Note that the tax reduces the returns from retained shares for the entrepreneur to $(1 - \tau)q\bar{e}$. Further, it reduces the portfolio investors’ willingness-to-pay for the company to $(1 - \tau)p_e$.

We now characterize the equilibrium that emerges after taxation, assuming that this equilibrium is still characterized by $e = \bar{e}$. For a given tax rate $\tau$ there exists at most one $q(\tau)$ such that (46) is binding. Making use of (47) in (48) shows: If the optimal contract $(q, p)$ is characterized by high effort, then, for a marginal increase in $\tau$ it holds that

$$\frac{dq^*(\tau)}{d\tau} = \frac{q^*(0)}{1 - \tau},$$

implying that $q^*(\tau) = q^*(0)/(1 - \tau)$.

The result in (49) is reminiscent of the Domar-Musgrave effect. Investors in the capital market are indifferent as regards buying a share in the firm’s profits at a price of $(1 - \tau)p_e$. The firm profit is proportionally reduced by the tax, but it is still stochastically independent from the rest of the capital market; this explains the strictly proportional reduction in the price, if the investors can still be sure that the entrepreneur will choose high effort. To establish this, an adjustment of the retained share in the company according to (49) is required.

We also note from (49) that the adjustment of the retained shares in the company reaches an exogenous constraint once the tax rate increases. This is the case when $\tau > 1 - q^*(0)$. At this point the entrepreneur needs to retain more than 100 percent of the firm to convince the portfolio investors that the entrepreneur will choose high effort. But even worse, if the government implements a tax rate for which $\tau > 1 - q^*(0)$, this implies that the entrepreneur will choose $e = 0$ even if he retained 100 percent of the firm. The outcome in this case will be an equilibrium with $e = 0$, in which the entrepreneur sells a share $1 - q = 1$ of the firm for a price of $p = p_0$.

We can summarize these results intuitively. Let us start from an equilibrium in which the entrepreneur is incentivized to expend high effort and retains a share in the company

\footnote{More formally, (49) can be obtained by totally differentiating (46) at the point of equality for $q$ and $\tau$.}
for this purpose, and assume that the introduction of a small proportional tax does not change the nature of the equilibrium. In this case, profit taxes reduce the entrepreneur’s incentives to provide high effort. In order to re-establish a sufficiently strong incentive to induce high effort, the share retained by the entrepreneur needs to be increased. The amount of this increase is similar to the Domar-Musgrave effect. The after-tax risk assumed by the entrepreneur in the equilibrium is precisely the same as in the equilibrium without a tax, and is independent of the level of taxation. It is just the amount of risk-taking needed to incentivize the entrepreneur appropriately and is determined by an incentive compatibility constraint. It is true that the government can consolidate some risk inside the government tax revenue. But this amount of risk consolidation just nets out with the adjustment in the risk-taking position of the entrepreneur.

Note that the arrival at (49) precisely hinges on the assumptions we made about the redistribution of the tax revenues. We redistributed these in such a way so as to compensate for the loss in income which the entrepreneurs would suffer in the face of taxation. If the tax proceeds are redistributed in a different fashion, much like in the context of Section 3, we would find income effects that may compound or counteract the reaction $\frac{dq}{dt}$ in (46). Accordingly, a profit tax is completely ‘neutral’ in a deeper sense if it does not fundamentally replace the incentive equilibrium that induces high effort: it leads to precisely the same risk allocation net of taxes as in the situation with a tax rate equal to zero. The tax does not improve or deteriorate the situation, and given the way of redistribution we have chosen, it does not affect the net wealth positions of any of the agents in the economy.

**Robustness considerations**  We have seen that this result is limited to the consideration of small tax rates. If the tax rate becomes large, the nature of the equilibrium changes. For larger tax rates, the choice of a contract with $q > 0$ and $p = p_\epsilon$ and with $\epsilon = \bar{\epsilon}$ is no longer optimal for the entrepreneur. Rather, the entrepreneur is better off by choosing an outcome with $q = 0$, $p = p_0$ and $\epsilon = 0$. Of course, compared to the incentive equilibrium for $\tau = 0$, this typically implies a reduction in aggregate welfare, as it makes all entrepreneurs worse off in terms of expected utility.

At first glance, the idea that the state can increase welfare by way of tax-induced risk-taking in a situation in which, due to asymmetric information, the market only achieves incomplete risk consolidation appears tempting. The state taking over risks, however, also leads to a restriction of the possibilities for entrepreneurs to credibly induce a high effort and makes this a credible choice in the beliefs of the capital market investors. The incentives created by the capital market to solve the principal-agent problem are thus undermined. A low tax rate is ineffective and a tax at a high enough rate is counterproductive. Instead of leading to an increase in welfare improvement, the entrepreneur’s utility is even diminished.

A further caveat applies if we consider an unexpected increase in the tax rate in an economy which already has a shareholdership structure that is optimally adjusted to a given profit tax rate. In this case it would have been desirable if the entrepreneurs had chosen a higher retained share in anticipation of the tax. But an ex-post adjustment is rather difficult.
Recall that the entrepreneur’s retained share equilibrium loses more in attractiveness due to the tax than the equilibrium without retained shares. This is an additional reason why the introduction or increase of a tax can lead to a transition from an equilibrium with positive retained shares and a high effort level to the moral-hazard equilibrium without retained shares and a complete sale of the company.

The analysis was simplified by the assumption that there are only two levels of management effort. This is an important assumption that is also used in Keuschnigg and Nielsen (2003, 2004). As shown by Hagen and Samarnes (2007), the equilibrium in a framework with a choice of a continuous effort reacts differently to the introduction or the increase in the tax: a higher tax will typically reduce the equilibrium effort that results in the equilibrium. Intuitively, additional effort increases the value of the project. By way of taxation the government participates in the additional value generated by additional effort, but not in the effort cost. Hence, the tax drives a wedge between the net return of the marginal unit of effort and the marginal cost of this effort, and this explains the reduction in equilibrium effort. The contractual relationship between the entrepreneur and the portfolio investors adds complexity to this problem, but does not fundamentally remove the distortionary effects for effort choice.

It can be concluded that the incomplete diversification of unsystematic risks to private capital markets may be attributed to corporate incentive problems. Many of the efforts brought to the table by the entrepreneur or manager operating the firm cannot be contractually defined, because only this person knows exactly how he or she contributes to the success of the firm. Well-diversified holdings of shareholders lead to a circumstance in which the firm’s owner-manager lacks the incentive to make an entrepreneurial effort. In a firm which has completely diversified ownership, no sufficient incentive exists due to the public-good-quality of management control on the part of the firm’s ownership to invest in the supervision of the firm and its management. A (second-best) solution to this incentive problem requires the firm to abstain from complete risk diversification. A shareholder with a qualified stake in the firm has at least a certain incentive to supervise it, and a manager in the firm who owns a substantial share of the company will also work toward bringing it to success.

The state’s proportional participation in the firm’s revenue by way of a proportional tax revenue reduces, with given participation rates, the incentive to good management and control of the firm. This effect is entirely analogous to an increase in share diversification. An owner with a 60% stake in the company who must pay a 50% revenue tax has only the same incentive as an owner with a 30% stake and no obligation to pay taxes. The tax means that a higher percent amount of the firm’s yield risk is diversified than would be in the laissez-faire case. The advantage of additional diversification would, however, be counterbalanced by the disadvantages associated with a reduction of the effort put into the supervision of the company and its management.

The market can react to such a tax through an appropriate alteration of the participation rates and thereby counteract the damaging effect of the tax. The market’s opportunities to contain the negative welfare effects of a tax are limited, however. A sufficiently high tax
necessarily leads to suboptimal incentives from suboptimal participation rates and to welfare losses due to changes in the incentive structure. A proportional tax on revenue thus proves itself to be, in this context and in the best case scenario, ineffective and innocuous. If it is in fact effective, it would also be harmful.

4.2 Adverse Selection

Another reason for incomplete risk diversification in private capital markets is adverse selection. Entrepreneurs with companies may want to go public, or entrepreneurs who have a given project may try to fund this project with equity. The initial owners of the companies or projects typically have a better knowledge of the profitability of their projects than outside investors. If entrepreneurs succeed in obtaining equity funding, we may distinguish between two prominent types of market equilibrium. One equilibrium has different prices for good and for bad projects or companies. The other has the same capital market prices for shares in good and bad companies/projects. The effect of taxation depends on the nature of the equilibrium. In what follows we do not provide an extensive analysis, we only survey some of the results and provide some intuition for the results that are an application of standard reasoning as outlined in insurance markets (Rothschild and Stiglitz, 1976).

Consider a large number of entrepreneurs, each of whom tries to get external equity funding for a company or a risky project. Suppose that each of these projects can either be successful and generate a positive surplus, or be unsuccessful and generate some loss. Suppose further that some projects are more likely to be successful than others. In this case, the fair market value of a good project is higher than that of a bad project. But the investors who could provide equity funding for such a project may face an information problem: they cannot observe the quality of the project. In an equilibrium, this may lead to a situation in which all projects get funding for the same price per project share. Under other conditions the entrepreneurs who own better projects may find a way to successfully signal this information to the investors who provide equity funding for these projects. If capitalists pay the same average price for good and bad risky projects, in the absence of taxation, the entrepreneurs of good firms are confronted with a dilemma when selling their company shares. They could sell the project on the capital market and benefit from the risk diversification taking place there. However, if they have a good project, the market will pay them less than the true expected project value. In this case, the owner of a good project typically chooses to sell only a proportion of his company and keep a share of the company. Owners of bad projects sell off the whole project. As a result, the capital market allocation has less risk diversification than what would be feasible. If the government enters into such a situation, through the tax, the state takes over part of the returns (or losses) of the risky ownership share that is kept by the owners of good projects. Just like in the Domar-Musgrave model, this reduces the net-of-tax riskiness of the owner’s asset holdings. Abstracting from possible income effects here, this incentivizes him toward increasing his ownership share of his company. In the general equilibrium, this effect results in a change in the composition of the market portfolio:
bad companies are still completely sold off in the capital market. But only a smaller share of the good companies is sold to the capitalists. The fair market price for this mixture of companies is lower. It becomes accordingly less attractive for founders of good firms to sell shares in the capital market. This price effect reinforces their willingness to retain shares in the good projects. Taxation of risky assets then leads to even less risk diversification in the aggregate.

If the owners of projects manage to signal the true quality of their projects to the capitalists, who may purchase shares in these projects, then taxation and signaling may interact in interesting ways. For instance, if the owners of good projects retain a large share in their project, this may credibly signal to the portfolio investing capitalists that the respective project is actually good and may lead to an equilibrium with higher market prices for shares in good projects than in bad projects. Taxation now affects the amount of risk-taking (net of taxation) for the owners of good projects. In order to provide a credible signal, they may have to expand their gross risk-taking, similar to the increased risk-taking described by Domar and Musgrave. Overall, as shown in Konrad and Richter (1995), the effect of additional risk consolidation inside the government tax revenue and the effect of an increase in the gross ownership share that is retained by the owners of good projects, does not net out completely. The additional risk consolidation dominates and the tax can therefore improve the allocation of risk.

These considerations assume that entrepreneurs find portfolio investors who purchase equity shares in these companies. Alternatively, investors may provide credit, or a mixture of both equity and credit.

The nature of problems that emerge in the context of outside funding more generally has attracted a great deal of attention in the theory of corporate finance. Many types of unobservability can make outside funding a serious problem. To these belong the unobservability or non-verifiability of managerial effort, the unobservability of choice behavior, for instance, the choice of the risk characteristics of the entrepreneurial project, and the choice of the long-term characteristics of the project. Other problems emerge in the context of the measurement of the project success. Actual firm profit may be difficult or costly to verify for the party which contributes to outside funding, or actual profit may even be completely unobservable. These problems can be addressed within the general approach of contract theory, and a wealth of results has been generated (see, e.g., Tirole 2006, Chapter 3 for an overview). Profit taxes or other types of taxes on capital return change the parameters of these problems. Taxes tend to withdraw resources from these markets for entrepreneurial activity. As the lack of sufficient inside funding is the starting point for this whole literature, there is a natural tendency that such a withdrawal of resources from the system may make existing problems more severe. However, the picture that emerges in studies of this problem is more complex and the answers are less unidirectional. The literature on the role of taxes in models with outside financing constraints is less extensive than the literature on outside funding capacity itself, but it is still considerable and too rich to give a survey here. Central to these papers was usually not the question of the effect of the tax on the risk allocation,
but the question of overinvestment in projects with profitability that is too low or underinvestment in projects with high profitability (see, e.g., DeMeza and Webb, 1987, 1988, 1989, Innes, 1991, and Fuest and Huber, 2007). An approach that takes on board many of the relevant aspects is by Boadway and Keen (2006).

5 Summary

In economics, the risk consolidating role of the state has been emphasized in many contexts, namely by referring to the results of the partial analysis of the Domar-Musgrave model. The popular proposition goes as follows: The state becomes a silent partner in many risky investment projects through the taxation of revenues. It thus reduces the risk that was taken over by the investor and enables the investor to take on additional risks. In addition to this, there is a common view that the laissez-faire economy is often affected by a tendency of underinvestment on risky projects. Hence, the tax would lead to an increase in risk-taking (gross) which would be desirable from an overall economic point of view. This view is not only well-represented in the literature in the context of human capital investment and income taxation (Eaton and Rosen, 1980a, 1980b; Varian, 1980; Hamilton, 1987; Wigger and von Weizsäcker, 1998), but it can also be found in the general economic discussion on taxes and risk-taking.

This paper surveys and revisits the theory on taxes on risky returns that originated from Domar and Musgrave (1944). Emphasis is given to the role of complete capital markets and on capital market imperfections arising from limited liability, moral hazard and adverse selection. The discussion on the effect of profit taxes in the general equilibrium with complete capital markets or complete risk markets shows that the popular proposition that is based on the Domar-Musgrave effect is not sound, neither in a normative nor in a positive respect. In particular, we survey results on super-neutrality of taxes on risky returns for the aggregate risk taking in an economy that are in sharp contrast with this proposition. However, we also find that taxes on risky returns can ameliorate problems that emerge from excessive risk taking if excessive risk taking is caused by limited liability rules. This effect emerges both in the partial analysis and in general equilibrium.

If capital markets – due to moral hazard or adverse selection - are incomplete and/or imperfect another channel for tax effects becomes important: In this case private market participants often refrain from eliminating the existing idiosyncratic risks through portfolio diversification in order to overcome incentive problems resulting from asymmetric information. When the state then intervenes with a tax and generates aggregate tax revenue with very different risk characteristics than the revenue from an individual portfolio thus consolidating risks this will also affect the incentive compatibility conditions which are required to solve control problem in the market allocation. As a reaction to the tax, and to reinstall proper incentives and to cope with the information problems the private agents may increase their holdings of idiosyncratic risks which may entail neutrality also in this situation. But if tax rates are high this possibility of self-correction is undermined or even destroyed so that
welfare losses are generated by the tax. In general, there is a fundamental conflict between consolidation of risks through the tax system on the one hand and preserving incentives in the case of asymmetric information on the other. Since the state is confronted with the same information problems as the market it cannot be expected a priori that the state will be able to replace the market and to cure its deficiencies when there is moral hazard or adverse selection (Hellwig, 1986, p. 247).

One observation is obvious from these considerations: the point of view in which the state consolidates risks, and thereby increases welfare, is a good deal more problematic than it may appear.

References


