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Fragmented Property Rights and Incentives for R&D

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Where product innovation requires several complementary patents, fragmented property rights can limit firms' willingness to invest in R&D. We consider the research intensity in multiple simultaneous R&D contests and how it depends on whether firms already hold relevant patents as well as the availability of an option to invent around. A measure of technological uncertainty is also analyzed. The multiple patent product involves an important hold-up problem that can reduce the overall R&D effort. Invent-around options moderate this problem. We also analyze targeted equilibria in which the aim of R&D can be to hold up a rival.

Key words: fragmented property rights; patents; contests; hold-up; R&D; inventing around; innovation
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1. Introduction

Many modern goods are produced using multiple, complementary technology components, which are often protected under a number of patents. This complementarity in production and in intellectual property generates several challenges. Heller and Eisenberg (1998) suggest that fragmented property rights defined around gene fragments in biotechnology may reduce firms' incentives to invest and commercialize products:

> Foreseeable commercial products, such as therapeutic proteins or genetic diagnostic tests, are more likely to require the use of multiple fragments. A proliferation of patents on individual fragments held by different owners seems inevitably to require costly future transactions to bundle licenses together before a firm can have an effective right to develop these products.
> (Heller and Eisenberg 1998, p. 699)

Their considerations received considerable attention, e.g., in the National Research Council report by Merrill et al. (2004), as they point at a potentially very serious problem if patents exhibit complementarity. Thumm (2005) conducted a survey among Swiss biotechnology firms, finding evidence that the existence of patents sometimes holds up the development of applications such as medical tests. Cho et al. (2003) conducted a large survey among biomedical laboratories in the United States. They find that "[e]mpirical evidence shows that the complementarity of patents indeed creates hold-ups and prevents the innovation of products, particularly in biomedicine" (Cho et al. 2003, p. 3). Arundel and Patel (2003) report the findings from a 1997 survey of the biotechnology sector in Canada in which 16% of firms reported that they had to abandon research projects because of patents held by rivals. Such abandonment would seem to indicate that "invent arounds" were not viable. Indeed, Encaoua et al. (2006) indicate that inventing around patents in biotechnology is difficult if it is the upstream research tools that are patented.

Biotechnology is generally an area in which there is strong complementarity between fundamental and applied research (Joly and de Looze 1996). Here Eisenberg (1996) suggests that hold-up problems may arise if the patenting of research tools hinders downstream development. With genetic material, Willison and MacLeod (2002) note that inventors of downstream applications often need to gather the rights of several patents. This is endorsed by Arora et al. (2001, p. 161), who consider the dependence of "gene-chip" producers such as Affymetrix or Nanogen on the development and use of new technologies that can be used to test for different drugs and pathologies.

Kingston (2001) suggests that the fragmentation of intellectual property rights can be especially damaging in the case of complex technologies:[1]

> ...if competing firms hold patents on different components of a complex technology, and they fail to cross-license them (which can happen from many causes, not

[1] The definition of a complex technology is "a process or product that cannot be understood in full detail by an individual expert sufficiently to communicate all details of the process or product across time and distance to other experts" (Roycroft and Kash 1999, p. 262).

> all of them rational), development in an entire industry can be slowed down or even rendered impossible.
> (Kingston 2001, p. 408)

Furthermore, Kingston (2001) states that delay in gaining access to all necessary components of a complex technology will be especially harmful for technologies with a short life-cycle. This will also be the case for nascent industries such as nanomedicine. Bawa (2005) indicates the emergence of a patent thicket in the development of single-walled carbon nanotubes that may slow down progress in the field of nanomedicine; this thicket involves IBM, NEC Corporation, and Carbon Nanotechnologies Inc. Lux Research (2005) examined patents in nanotechnology generally and found that the intellectual property for the nanomaterial groups' dendrimers and quantum dots is particularly entangled.

For network industries, Rahnasto (2003) argues that simultaneous innovation has led to a wide range of products based on combinations of patents that are held by multiple owners. Citing this case, Kultti and Takalo (2008) state that the hold-up power conferred to a single patent holder retards the use of innovations.[2] Arora et al. (2001, p. 266) identify software, semiconductors, and computers as industries in which "the opportunities for hold-up are enormous" because of the fragmentation of intellectual property. Perhaps the most famous hold-up cases that have occurred in multicomponent products are airplanes and radio at the beginning of the 20th century. These problems were solved by the outbreak of the First World War (see Merges and Nelson 1992 and Schaafsma 1995).

In this paper, we focus on the incentives to acquire patents that yield some of their commercial value by being used jointly in a multicomponent application, as discussed by Heller and Eisenberg (1998). We consider two cases: (i) symmetric firms that start with no initial stock of patents and (ii) asymmetric patent races between firms that may have acquired some patents already. From the latter case it becomes transparent that firms acquire patents both to ensure the intellectual property rights for innovation and for the option of holding-up other firms. Outcomes in which several firms win some of the patents become very likely.[3]

We allow for market uncertainty (as defined in Loury 1979) and an important type of technological uncertainty and consider how this uncertainty affects the equilibrium outcome. We also vary the degree of complementarity of patents by allowing them to be useful in isolation and to yield an additional commercial value if used jointly in a multicomponent application. Adjusting the relative size of these commercial values facilitates an analysis of the specific role of complementarity. We also take into consideration that firms may have an option to invent around a given patent at some cost and may trade patents, once the patents are awarded to them.[4] Once this exchange of intellectual property rights is completed, owners use their respective patents to produce and to make profits in a product market. As discussed by Shapiro (2001), several trading regimes can be distinguished. Because the exact nature of trading is not our main focus, we assume that firms can freely trade exclusive rights to use single patents. Exchange of rights will lead to an ex post efficient allocation of patents.[5]

Patent races and the role of market structure for R&D have received much attention in the literature as well, starting with Arrow (1962). In this context much of the focus is on single patents that may be attained in an R&D contest that may have one or multiple stages, and to the asymmetry between an incumbent and a challenger if they compete for a new, superior patent. Recently, Fershtman and Markovich (2006) consider multistage patent races with different rules about the stage at which the relevant patent is granted. The interaction of multiple patents and their complementarity has received less attention.[6]

We describe the analytical framework of a game with three stages in §2. We set the stage for an analysis of the patent race in §3. In §4, we characterize the equilibria of the patent race. The analysis in §5 looks at the possibility of targeted equilibria in which firms explicitly aim to hold each other up by winning patents for use as bargaining chips. In §6, we consider an option to "invent around" a given patent. We discuss these results and conclude in §7.

[2] In other work, Kultti et al. (2006) cite the case of Nokia and Kyocera Wireless, which were involved in a two-year patent dispute that culminated in a cross-licensing agreement.

[3] For an analysis of the empirical relevance of complementarities in internal and external R&D activities and its determinants, see Cassiman and Veugelers (2006). Although the focus of their analysis is different, the empirical result that 66% of innovating firms rely on both their own R&D and on external R&D is in line with our theoretical findings.

[4] It is also interesting to consider incentives to negotiate this trade prior to the patent race, for instance, by formation of a patent pool. The trade-offs between ex ante and ex post bargaining about patent rights are analyzed by Siebert and von Graevenitz (2006).

[5] Shapiro (2001) and Lerner and Tirole (2004) also consider problems with patent thickets, patent overlap, and complementary patents; their main focus is inefficient use of an existing, exogenously given set of patents. In contrast, we address the problem of endogeneity of patenting, particularly the case of multiple patent races for complementary patents.

[6] Some work in this context focuses on a sequential structure of innovation, the incentives to expend R&D effort in a first or in a second stage of a cumulative innovation, and its policy implications for the allocation rules for the trading of patents (e.g., Scotchmer 1996, Green and Scotchmer 1995, Denicolo 2002).

2. The Analytical Problem

Consider two firms A and B that compete with each other in the following three-stage game.

In Stage 1 the two firms spend efforts on R&D. They already hold $k = k_A + k_B$ patents, of which firm A holds $k_A \geq 0$ and firm B holds $k_B \geq 0$. They need to innovate and patent n further essential components of a new good. Each patent i is awarded as the outcome of an independent patent race, in which $x_i \geq 0$ and $y_i \geq 0$ are the efforts expended by firms A and B in the patent race i, respectively. These effort choices are made simultaneously. We define $\mathbf{x} \equiv (x_1, \ldots, x_n)$ and $\mathbf{y} \equiv (y_1, \ldots, y_n)$.

The probability that firm A gets the relevant information about component i prior to firm B and wins the patent i is a function $p_i(x_i, y_i)$ of the efforts x_i and y_i, and the respective probability for B is $p_i(y_i, x_i)$, where symmetry between the firms is applied. Technological uncertainty about whether the patent is feasible may enter in different ways. We allow for two types of technological uncertainty. First, there is a threshold level of effort, denoted by θ. If a firm exhibits an effort less than θ for patent i, it does not uncover the relevant information that is needed to apply for a patent. In general, θ can be drawn from a distribution, but we will consider deterministic θ. We will compare the case of low thresholds and cases with high threshold, as they lead to structurally different equilibrium outcomes.

Second, we allow for uncertainty about whether an invention i can be made at all, and the probability for this is denoted by γ. More specifically, the probability that A wins patent i is

$$p_i(x_i, y_i; \gamma, \theta) = \begin{cases} \gamma \dfrac{x_i}{x_i + y_i} & \text{if } x_i \geq \theta > 0 \text{ and } y_i \geq \theta > 0, \\ & \quad \text{or } x_i > \theta = 0 \text{ and } y_i > \theta = 0, \\ \gamma & \text{if } x_i \geq \theta > 0 \text{ and } y_i < \theta, \\ 0 & \text{if } \theta > x_i \text{ or } x_i = 0. \end{cases} \tag{1}$$

According to (1), an innovation is feasible and can be patented with a probability $1 \geq \gamma > 0$ and is technologically not feasible with the remaining probability $1 - \gamma$. If the innovation is feasible, an innovation effort that is positive and at least θ is required to participate in the innovation race. If more than one firm enters the race with a sufficient innovation effort of at least θ, then $x_i/(x_i + y_i)$ is the probability that A wins the patent if it is technologically feasible; if only one firm makes the necessary investment, it wins with certainty if the patent is feasible.

In reality, the threshold θ may also be a random variable, such that the probability that a given amount of effort is sufficient for the desired technological breakthrough is increasing in own effort. As such a framework is analytically not tractable, we capture the random aspect of technological uncertainty with γ and the aspect that a substantial probability for a technological breakthrough may need substantial effort by θ. The idea behind a required threshold level of investment for innovation is conveniently summed up by Thornhill (2006) as a minimum efficient scale in the domain of firm knowledge. He states,

> In high technology industries where the pace of technical change is high, new products may have to overcome a significant hurdle to distinguish themselves from the offerings of competing firms. …When the pace and magnitude of change is less extreme (in more stable industries) innovations may not require the same degree of "newness" to be successful. (Thornhill 2006, p. 688)

This effectively delineates that case of high θ, which is the theme of §5, and the case of low θ, which appears in §4.

For $\gamma = 1$ and $\theta = 0$ Equation (1) corresponds to the case with pure market uncertainty as coined by Loury (1979).[7] Further, this description of a firm's market uncertainty in the R&D contest between two firms can be justified using an important equivalence result that has been developed by Baye and Hoppe (2003).[8] They show that many types of innovation contests and patent races in which the process of innovation follows a stochastic process can be represented equivalently as a simple lottery contest. We assume that p_i and p_j in (1) are stochastically independent of each other ($i, j = 1, 2, \ldots, n$, $i \neq j$).

At the beginning of Stage 2 the set of patents is awarded. We define the vector $\mathbf{z} = (z_1, \ldots, z_k, z_{k+1}, \ldots, z_{k+n})$, describing the *pretrade* ownership structure of patents with $z_i = a$ if firm A holds patent i, and $z_i = b$ if firm B holds patent i, and $z_i = 0$ if neither has attained patent i at the end of the patent race. Firms can negotiate with each other and trade patents accordingly. The joint use of all $n + k$ patents in the multicomponent good by a monopolist yields a monopoly rent M. In addition, each patent yields value $R \geq 0$ in its own right, for instance, in other applications. To produce the multicomponent good, a firm needs to be in control of all $n + k$ patents. This means that the firm can use the respective technology covered by this patent without any risk of being

[7] Except that we assume that the patent is not awarded if both players expend zero effort, instead of assuming a probability of 1/2 in this case. For the equilibrium outcomes in Propositions 1, 2, and 3, this behavior at $x_i = y_i = 0$ does not matter.

[8] For applicability of this function for the limit of a low discount rate, see also Nti (1997). The function also has been axiomatized for contests more generally by Skaperdas (1996).

sued by the actual holder of the patent, either because the firm itself is the holder of the patent or because the outcome of trading and negotiating between firms leads to this security.[9]

Free trade governs trade in our model, and this is defined as follows. Firms A and B may trade the exclusive ownership of each single patent. The *post-trade* ownership structure of patents is given by some $\hat{\mathbf{z}} = (\hat{z}_1, \ldots, \hat{z}_k, \hat{z}_{k+1}, \ldots, \hat{z}_{k+n})$, with $\hat{z}_i \in \{a, b, 0\}$ and a transfer payment from A to B of size $\phi(\mathbf{z}, \hat{\mathbf{z}})$, which can be any real number that they agree on. Of course, only existing patents can be traded; i.e., $\hat{z}_i = 0$ if $z_i = 0$.

At the beginning of Stage 3, patent negotiations are completed. Patent rights are finally allocated between firms. Each patent can be used independently, and a firm that is in control of all $k+n$ patents can produce and market the multicomponent good. If neither firm controls all rights, neither can produce the multicomponent product, and both firms earn only the profits according to the independent values of their patents. Denote n_A and n_B the numbers of patents eventually owned and controlled by A and B, respectively. If $\hat{\mathbf{z}} = (a, \ldots, a) \equiv \mathbf{a}$, then firm A is in control of all patents and receives the monopoly profit $M + n_A R$, and firm B receives zero profit, and vice versa if $\hat{\mathbf{z}} = (b, \ldots, b) \equiv \mathbf{b}$. By symmetry, the monopoly rent from the multicomponent product, M, does not depend on which firm is the monopolist. If $\hat{\mathbf{z}} \notin \{\mathbf{a}, \mathbf{b}\}$, then the multicomponent product is not produced. Each firm has its own rights in a number of patents, n_A and n_B, and receives $n_A R$ and $n_B R$, respectively.

Summarizing, the market profit of firm A is $M + n_A R$ if $\hat{\mathbf{z}} = \mathbf{a}$ and $n_A R$ otherwise; firm B gets $M + n_B R$ if $\hat{\mathbf{z}} = \mathbf{b}$ and $n_B R$ otherwise. Firms' expected payoffs are therefore

$$\pi_A = \begin{cases} \phi(\mathbf{z}, \hat{\mathbf{z}}) + M + n_A R - \Sigma_i x_i & \text{if } \hat{\mathbf{z}} = \mathbf{a}, \\ \phi(\mathbf{z}, \hat{\mathbf{z}}) + n_A R - \Sigma_i x_i & \text{if } \hat{\mathbf{z}} \neq \mathbf{a} \end{cases} \tag{2}$$

and

$$\pi_B = \begin{cases} -\phi(\mathbf{z}, \hat{\mathbf{z}}) + M + n_B R - \Sigma_i y_i & \text{if } \hat{\mathbf{z}} = \mathbf{b}, \\ -\phi(\mathbf{z}, \hat{\mathbf{z}}) + n_B R - \Sigma_i y_i & \text{if } \hat{\mathbf{z}} \neq \mathbf{b}. \end{cases} \tag{3}$$

The payoff consists of the compensation $\phi(\mathbf{z}, \hat{\mathbf{z}})$ received by A and paid by B when patent rights are reallocated in Stage 2, plus the monopoly rent in case the firm controls all patent rights, minus the firm's aggregate cost of the effort that has been spent in the patent competition.

[9] Dynamic commitment issues that emerge in sequential R&D, or uncertainty about a possible infringement when commercializing a new product, are absent from our consideration.

3. Patent Trade and Profits

Solving this game by backward induction, no decision has to be analyzed at Stage 3. The posttrade ownership structure $\hat{\mathbf{z}}$ fully determines the payoffs in the market game, as characterized by (2) and (3).

Consider Stage 2. The pretrade ownership structure $\mathbf{z}$ is given at the beginning of Stage 2. Three cases may emerge: (i) Let $z_i = 0$ for some $i \in \{1, \ldots, n\}$. Then there is no need for trade at this stage, because the multicomponent product cannot be made. (ii) Let $z_i \neq 0$ for all $i = 1, \ldots, n$, and $\mathbf{z} = \mathbf{a}$ or $\mathbf{z} = \mathbf{b}$. It follows from subgame perfection that any bargaining outcome will yield the monopoly profit to the firm that owns all patents at the beginning of Stage 2. (iii) Let $z_i \neq 0$ for all $i = 1, \ldots, n$, but $\mathbf{z} \notin \{\mathbf{a}, \mathbf{b}\}$. Each firm holds at least one patent and the other firm holds all others. Efficient negotiations will generate a joint profit of $M + (n+k)R$ in total. The benefit from trading is divisible and does not depend on the number of patents that each firm holds; this benefit is equal to M.

Many bargaining concepts that could be used to determine the distribution of this surplus yield the same outcome, given the assumption about risk neutrality of firms and symmetry. For simplicity, we assume symmetric Nash bargaining among risk-neutral firms. Let firms have profits $n_A R$ and $n_B R$ as their outside options. Then the surplus from efficient negotiations is the monopoly profit M, and they share this evenly in the Nash bargaining solution. Hence, if $\mathbf{z} \notin \{\mathbf{a}, \mathbf{b}\}$, then either $\hat{\mathbf{z}} = \mathbf{a}$ and $\phi(\mathbf{z}, \hat{\mathbf{z}}) = -M/2 - n_B R$ or $\hat{\mathbf{z}} = \mathbf{b}$ and $\phi(\mathbf{z}, \hat{\mathbf{z}}) = M/2 + n_A R$, both yielding payoffs equal to $\pi_A = M/2 + n_A R - \Sigma_i x_i$ and $\pi_B = M/2 + n_B R - \Sigma_i y_i$.

4. Covering All Bases—Symmetric Equilibria in the Patent Race

We now turn to solving the multipatent R&D contest in Stage 1, given the payoffs from equilibrium play in Stages 2 and 3 as in §3 and also taking into consideration technological uncertainty. We can state the properties of the subgame perfect equilibrium for three different cases, depending on whether some firm already holds some patents. The threshold level θ turns out to have strong implications regarding the nature of R&D equilibrium, and in this section we consider the outcome of competition if the minimum effort θ that is needed to be a relevant player in the respective patent race is low. Analytically we focus on the case with $\theta = 0$ and explain how the results extend to a large set of θ values at the end of this subsection.

Proposition 1. *Let $\theta = 0$. If $k = 0$, the symmetric subgame perfect equilibrium of the multipatent contest with unconstrained bargaining between two firms exists and* *is described by an interior symmetric equilibrium in the simultaneous contests for n patents with efforts*

$$x_i^* = y_i^* = \frac{\gamma^n M}{2^{n+1}} + \frac{\gamma R}{4} \quad \text{for all } i = 1, \ldots, n \tag{4}$$

and equilibrium payoffs

$$\pi_A^* = \pi_B^* = \frac{\gamma^n M}{2}\left(1 - \frac{n}{2^n}\right) + n\gamma \frac{R}{4} > 0 \tag{5}$$

for a wide parameter range. A sufficient condition characterizing this parameter range is

$$\frac{4R}{M} \geq \gamma^{n-1} \frac{(n-3)}{(n+1)}. \tag{6}$$

Proof. We consider Stage 1 and use the properties of the equilibrium in the continuation game at Stage 2. Consider A's objective function using $\theta = 0$ and $k = 0$. With an efficient allocation of patent rights in Stage 2 and an equal split of the benefits from a possible reallocation at that stage, the expected profit of firm A is a function of the two firms' contest efforts:

$$\pi_A = \gamma^n \frac{M}{2} + \prod_{i=1}^{i=n} p_i(x_i, y_i) \frac{M}{2} - \prod_{i=1}^{i=n} (1 - p_i(x_i, y_i)) \frac{M}{2} + \sum_{i=1}^{n} p_i(x_i, y_i) R - \Sigma_i x_i. \tag{7}$$

Each patent has a probability of γ for being feasible, making the multicomponent product technically feasible with probability γ^n. Firm A receives the monopoly rent M on this application if it wins all patents, no share of this rent if firm B wins all patents, and just $M/2$ if all patents are developed and each firm holds some of them. This explains the first three terms in (7). Firms also receive expected benefits from each single patent obtained, equal to the independent value of the patent times the probability γ. These benefits are summed up in the fourth term. Finally, the firm has to pay the sum of its efforts in the n parallel single patent contests, and this constitutes the fifth term on the right-hand side.

We use Lemma 1 from the appendix: if B chooses $\mathbf{y} = (y, y, \ldots, y)$, then the optimal reply by firm A is some $\mathbf{x}(\mathbf{y})$ with $x_1(y) = x_2(y) = \cdots = x_n(y) \equiv x(y)$, and vice versa. Using this result in (7) the first-order condition for A's reply to the uniform vector $\mathbf{y}$ as in (4) is the vector $\mathbf{x}(\mathbf{y}) = (x, x, \ldots, x)$ that maximizes

$$\pi_A(x) = \frac{\gamma^n M}{2} + \frac{x^n - y^n}{(x+y)^n} \frac{\gamma^n M}{2} + n\gamma \frac{x}{x+y} R - nx. \tag{8}$$

Note that

$$\frac{d\pi_A(x)}{dx} = n \frac{x^{n-1}y + y^n}{(x+y)^n (x+y)} \frac{\gamma^n M}{2} + n\gamma \frac{y}{(x+y)^2} R - n. \tag{9}$$

The first-order condition for B is obtained by replacing x by y and vice versa. These first-order conditions are simultaneously fulfilled for

$$x = y = \frac{\gamma^n M}{2^{n+1}} + \frac{\gamma R}{4}. \tag{10}$$

Note also that this is the only symmetric solution of these first-order conditions. Inserting this solution into (8) yields payoffs $(\gamma^n M/2)(1 - n/2^n) + n\gamma R/4$, which are always nonnegative. Note further that the payoff functions are concave if $d^2\pi_A(x)/(dx)^2 < 0$ or if

$$(n-1)(x^{n-2} - y^{n-2}) y^2 \gamma^n M - (x^{n-1}y + y^n) 2\gamma^n M - 4\gamma y (x+y)^{n-1} R < 0. \tag{11}$$

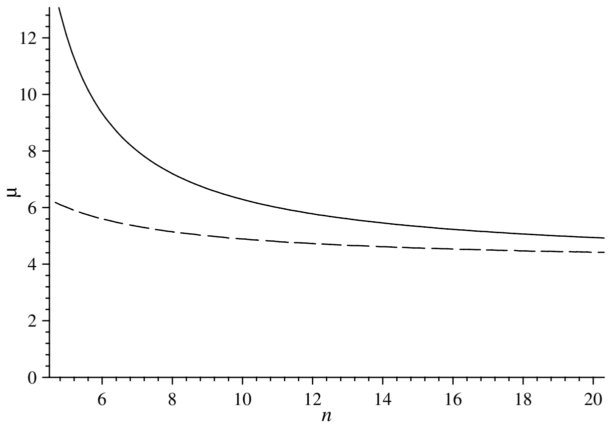
This condition is fulfilled for all $x \leq y$ but need not hold for all $x > y$ for combinations of M, R, and n.[10] The optimal x is bounded from above by the maximum size of the payoff from winning a single patent, which is smaller than or equal to $M + R$. For large n, the binomial term $(x + y)^{n-1}$ increases rapidly, whereas γ^n decreases rapidly, as $\gamma < 1$, making the range large in which (4) are globally optimal replies to each other. A lower bound of this range can be obtained as follows. For $x > y$ the left-hand side of (11) is strictly smaller than $(n-1)x^{n-2}y^2\gamma^n M - x^{n-1}y2\gamma^n M - 4\gamma y x^{n-1} R - 4\gamma x^{n-2} y^2 R(n!/[(n-1)!1!])$. This is obtained from (11) by dropping the negative terms $(-y^{n-2}y^2\gamma^n M)$ and $(-y^n 2\gamma^n M)$, using only the first two terms from the polynomial expression $(x + y)^{n-1}$, and dropping all other terms, and by making use of $x \geq y$ and replacing $(-x^{n-1}y)$ with the smaller term $(-x^{n-2}y^2)$. This yields the sufficient condition (6) as in Proposition 1. □

Proposition 1 characterizes the unique symmetric equilibrium that exists for a wide range of parameter values. The condition (6) is fulfilled for $n \in \{1, 2, 3\}$ for any ratio of M/R. For larger n, it is important to note that (6) is generally more restrictive than the condition of global concavity, (11), whereas concavity itself is more restrictive than what is needed for global optimality of (4). But even (6) encompasses all cases in which M is not disproportionately larger than the individual use of the patents and is also fulfilled for n sufficiently large if $\gamma < 1$.

Define $\mu \equiv \gamma^n M/\gamma R = \gamma^{n-1} M/R$, which is the ratio of the expected value of the multicomponent product to the expected independent value of a single patent. For $n \leq 3$ condition (6) always holds. For $n > 3$ the condition (6) can be written as $\mu \leq (4(1+n))/(n-3)$, which holds for parameter combinations (n, μ) in the

[10] In particular, in an earlier verison of this paper (Clark and Konrad 2006), we have shown that, for the extreme case with $R = 0$ and $\gamma = 1$, x and y as in (4) are globally optimal replies to each other, but only for $n < 7$.

Figure 1 Condition for Symmetric Equilibrium



Notes. All combinations of $\mu = \gamma^{n-1}M/R$ and n below the solid line fulfill the sufficient condition in Proposition 1. The dashed line refers to the respective condition in Proposition 2.

area in Figure 1 below the whole line. The ratio μ can also be interpreted as measuring the importance/strength of complementarity. It is high if the patents are primarily useful in the multicomponent product but are of little use apart from this application. Figure 1 masks the fact that μ depends on n, typically decreasing strongly in this parameter, as the multicomponent product needs n patents, each of which is subject to technological uncertainty.

The two firms' total effort in all patent contests sums up to

$$nx + ny = \frac{n\gamma^n M}{2^n} + n\frac{\gamma R}{2}. \tag{12}$$

The first term in (12) has its maximum for $n = 1$ and is strictly decreasing in n for $n \geq 2$. If the patents only serve as inputs for the multicomponent product, then a larger number of patents strongly reduces aggregate effort. Technological uncertainty strengthens this effect, as the probability will be high that one of the patents turns out to be infeasible. The second term in (12) accounts for the effect that each additional patent is useful in isolation. This term is increasing in n. If the acquisition of each single patent has an additional independent value, and if there are more patents, this increases the value of the sum of these independent values and firms will expend effort in each additional patent contest. The total effect of an increase in n is, therefore, typically not monotonic. For small n the first effect dominates. For large n, the first effect becomes insignificant, and the second effect dominates, unless $R = 0$.[11]

Technological uncertainty can reduce the total amount of effort in (12) that is induced by the competition between the firms, as such competition effort is useless with some probability, even if the other player does not expend effort. Technological uncertainty also compounds the problem of the feasibility of the multicomponent good, such that an increase in technological uncertainty (fall in γ) reduces the effort that is motivated by the potential gain from the multicomponent good more strongly than the effort that is motivated by the independent benefits from each single patent.

Finally, (12) shows that the problem of complementarity of patents in the multicomponent product becomes less important for total R&D if the complementarity of patents is weakened: the less valuable the complementary use, M, is compared to the sum of the independent values of the patents, nR, the more similar the problem becomes to a standard setup of a patent race.

The equilibrium probability by which all patents are gained by one firm only is a decreasing function in n, for two reasons. From symmetry and the binomial distribution, for $n = 2$, the probability is $\gamma^2/2$, i.e., $\gamma^2/4$ for each firm, and for $n = 6$, the probability that all patents end up with one firm is $\gamma^6/32$. This follows as the probability that all patents are technically feasible is γ^6, and, conditional on all patents being invented, each firm has an equilibrium probability of $(1/2)^6 = 1/64$ to win all of them. Hence, parallel contests for patents that are complementary and essential for an innovation typically lead to a situation in which firms rely on R&D results that are obtained by other firms. This result is in line with empirical evidence by Cassiman and Veugelers (2006), who find that 66% of innovating firms rely on both their own and other firms' R&D results.

We now turn to the situation in which one of the firms holds some patents already, prior to the simultaneous contests for n remaining patents. Without loss of generality, we assume that this firm is firm A.

Proposition 2. *Let $\theta = 0$. Suppose $k_A > 0$ and $k_B = 0$. A sufficient condition for a symmetric equilibrium in pure strategies in which*

$$x_i^* = y_i^* = \frac{\gamma^n M}{2^{n+2}} + \frac{\gamma R}{4} \tag{13}$$

is $n = 2$, or the condition $(n-1)\gamma^{n-1}M \leq 4(n+1)R$ for $n \geq 3$.

[11] A referee has pointed out that a recent ruling of the US Supreme Court, KSR International vs. Teleflex (April 30, 2007), has strengthened the obviousness criterion for patents. In our analysis this means that n, the number of inventions needed to be patented, is effectively reduced. This enhances the motive to carry out R&D to secure complementary patents and may be in this sense pro-innovation.

Proof. Suppose that firm A holds a positive number of patents. Consider the contest over the remaining n patents. When the ex post allocation of patents is resolved efficiently, as described in §2, the expected payoffs to the firms are

$$\pi_A = \gamma^n \frac{M}{2} + \prod_{i=1}^{n} p_i(x_i, y_i)\frac{M}{2} + \sum_{i=1}^{n} p_i(x_i, y_i)R + k_A R - \Sigma_i x_i \quad \text{and} \tag{14}$$

$$\pi_B = \left(\gamma^n - \prod_{i=1}^{n} p_i(x_i, y_i)\right)\frac{M}{2} + \sum_{i=1}^{n}(\gamma - p_i(x_i, y_i))R - \Sigma_i y_i. \tag{15}$$

Using that the optimal choices of efforts of one player are uniform along all patent contests if the other player chooses efforts uniformly (Lemma 1 in the appendix), the payoff functions reduce to

$$\pi_A = \frac{\gamma^n M}{2} + \frac{x^n}{(x+y)^n}\frac{\gamma^n M}{2} + \left(n\gamma\frac{x}{x+y} + k_A\right)R - nx \quad \text{and} \tag{16}$$

$$\pi_B = \left(1 - \frac{x^n}{(x+y)^n}\right)\frac{\gamma^n M}{2} + n\gamma R\frac{y}{x+y} - ny. \tag{17}$$

Firm B cannot win all $k+n$ patents, as A already holds $k = k_A$ of them. If firm B wins m patents, it earns the independent value mR of these patents. In addition, B can hold A from using its patents in the application that requires use of all $n+k$ patents, and negotiating with A would yield B an additional benefit of $M/2$ if $m \geq 1$, provided that A and B together hold all $n+k$ patents. The latter event occurs with probability γ^n. Interior solutions are characterized by first-order conditions

$$\begin{aligned} \frac{\partial \pi_A}{\partial x} &= \frac{nx^{n-1}y}{(x+y)^{n+1}}\frac{\gamma^n M}{2} + n\gamma R\frac{y}{(x+y)^2} - n = 0, \\ \frac{\partial \pi_B}{\partial y} &= \frac{nx^n}{(x+y)^{n+1}}\frac{\gamma^n M}{2} + n\gamma R\frac{x}{(x+y)^2} - n = 0, \end{aligned} \tag{18}$$

and x and y in (13) solve these equations. We show that payoffs are positive at the equilibrium values (13) and confirm that these solutions are globally mutually optimal replies.

The payoff for player B is higher for the candidate equilibrium efforts than for $y = 0$, given A's equilibrium choice of effort, as $(1 - 1/2^n - \frac{1}{2}(n/2^n))\gamma^n M/2 + n\gamma R\frac{1}{4} > 0$. For firm A, the payoff from $x = 0$ is $\gamma^n M/2 + k_A R$. The payoff from the candidate equilibrium reply is $\gamma^n M/2 + 1/2^n(\gamma^n M/2) + (n\gamma\frac{1}{2} + k_A)R - n(\gamma^n M/2^{n+2}) - n(\gamma R/4)$. The latter payoff is (weakly) higher than the former if

$$(n-2)\gamma^{n-1}M \leq n2^n R. \tag{19}$$

Note that this condition is always fulfilled for $n = 2$ but imposes a condition regarding the relative size of M and R for $n \geq 3$.

Turn now to global optimality. Note first that $\partial^2\pi_B/(\partial y)^2 < 0$ for all $x, y > 0$. The second derivative $\partial^2\pi_A/(\partial x)^2$ is negative if

$$((n-1)x^{n-2}y - 2x^{n-1})\gamma^{n-1}M - 4R(x+y)^{n-1} < 0. \tag{20}$$

For $x > y$, a sufficient condition for this is $(n-3)x^{n-2}y\gamma^{n-1}M < 4R(n+1)x^{n-2}y$, which is still weaker than the condition in the proposition. If $y \geq x \geq 0$, then a sufficient condition for (20) to hold is $(n-1)x^{n-2}y\gamma^{n-1}M - 4R(ny^{n-2}x + y^{n-1}) < 0$, which is obtained from (20) by disregarding a number of terms that enter negatively. By $y \geq x$, this condition is also weaker than the condition in Proposition 2.

Finally, note that the condition $(n-1)\gamma^{n-1}M \leq 4(n+1)R$ is also sufficient for (19) to hold for $n \geq 3$. □

The condition in Proposition 2 is only a sufficient condition for (13) to be globally optimal mutual replies. Using the ratio μ again in this condition, the dashed line in Figure 1 illustrates that even this condition is still fulfilled for a large range in which the expected value of the multicomponent application does not considerably exceed the expected independent values of patents, i.e., in situations for which complementarity is not too strong (recall that μ itself is typically decreasing in n).

If the condition (19) does not hold, the payoff for A from x^* given y^* as in (14) becomes negative, and the symmetric equilibrium ceases to exist. Intuitively, if the independent value of patents is close to zero ($R = 0$), then B has the same additional benefit of winning only one of the patents that A has from winning all the n patents. If B chooses some given effort in all n parallel contests, A's benefit of competing in the ith patent race depends on the probability of winning all other $n-1$ patent races as well. If n increases, it becomes increasingly expensive for A to keep this probability sufficiently high to make it worthwhile to compete for the ith patent. In short, for high n, it becomes too expensive for A to increase the win probability for all other $n-1$ patents sufficiently to make it worthwhile to compete for the ith patent, and so for all n patents. Firm A essentially gives up regarding the n additional patents. Focussing deterministically only on a subset of patents is also suboptimal for firm A, as it needs to win all patents to avoid the hold-up. Of course, simply giving up and choosing zero effort will not be feasible as an equilibrium. Firm A will expend positive effort with some probability, and this suggests that the equilibrium is in mixed strategies.

The precise nature of the equilibrium that applies if (19) is violated cannot be stated analytically in closed-form solutions for the general case with $\gamma < 1$, and $R > 0$ and $M > 0$. Intuition for the type of equilibrium that emerges in this case can be gained for the special case with $n > 2$, $R = 0$, and $\gamma = 1$. It can be shown that the leading firm may not gain from actively participating in the patent race for the n additional patents in this case and is indifferent between choosing zero effort and competing and trying to win all n patents.[12]

Finally, we consider the case in which each firm owns some patents at the outset.

Proposition 3. *Let $\theta = 0$. If $k_A > 0$ and $k_B > 0$, then the equilibrium efforts in the different patent contests are $x_i^* = y_i^* = \gamma R/4$, and the equilibrium payoffs are $\pi_A^* = \gamma^n M/2 + \gamma n R/4 + k_A R$ and $\pi_B^* = \gamma^n M/2 + \gamma n R/4 + k_B R$.*

Proof. Let $k_A > 0$ and $k_B > 0$. If $z_i = 0$ for some $i \in \{1, \ldots, n\}$, then the value in the multicomponent product will not be unlocked. Otherwise efficient negotiations will lead to monopoly profit, which is shared equally between the two firms. Accordingly, the firms compete for n independent prizes of size γR, and equilibrium in each of these contests is known to be unique and characterized by $x_i = y_i = \gamma R/4$ (see, e.g., Szidarovszky and Okuguchi 1997). □

If both firms already own some patents, they are already veto players regarding the bargaining and production stages, and the further allocation of the remaining n patents is not payoff relevant with respect to the market outcome for the good that is produced by the complementary use of all $k + n$ patents. Therefore, the complementary use of the patents does not affect the firms' bidding.

Note that the results for which a symmetric equilibrium with positive efforts x^* for all patents exist extends to the whole set of $\theta \in [0, x^*)$. This follows from the fact that choices x_i and y_i occur simultaneously. If $\mathbf{x}^*$ is firm B's optimal reply to $\mathbf{y}^*$ with $x_i^* \in [0, \infty)$, then $\mathbf{x}^*$ remains an optimal reply to $\mathbf{y}^*$ if a restriction on x_i is imposed that does not preclude the choice $x_i = x^*$. The same argument applies for firm B.

Note also that, if $\pi_A(\mathbf{x}^*, \mathbf{x}^*) > 0$ for an interior equilibrium with efforts as in (4) in Proposition 1, then a small positive ϵ exists such that $x_i = y_i = \theta$ are equilibrium efforts for $\theta \in (x^*, x^* + \epsilon)$. This can be shown using the symmetry properties of positive optimal replies in Lemma 1, together with the fact that, at $x_i = y_i = x^* + \epsilon$, we find that a general increase in all x_i by a marginal unit causes a change in A's payoff by

$$n\frac{(\gamma^n M/2^{n+1} + \gamma R/4)}{(\gamma^n M/2^{n+1} + \gamma R/4) + \epsilon} - n < 0,$$

implying that $\mathbf{x} = (\theta, \theta, \ldots, \theta)$ is locally a constrained optimal reply to $\mathbf{y} = (\theta, \theta, \ldots, \theta)$ for small ϵ. Continuity of π_A together with $\pi_A(\mathbf{x}^*, \mathbf{x}^*) > 0$ implies that $\pi_A(\boldsymbol{\theta}, \boldsymbol{\theta}) > 0$, making this vector $\boldsymbol{\theta}$ also a globally optimal reply.

We summarize this as

Corollary 1. *Let $\theta \in [0, x^* + \epsilon]$ with x^* defined in Proposition 1 and small positive ϵ. Then for $k = 0$ an equilibrium exists with uniform efforts $x_i = y_i = \max\{x^*, \theta\}$.*

Similar generalizations apply for the equilibria characterized in Propositions 2 and 3.

5. Targeted Equilibria—Patents as Bargaining Chips

The equilibria that we have studied so far are symmetric. We turn now to studying the possiblility of asymmetric, targeted equilibria.

For the case of a very small threshold, we cannot fully rule out asymmetric equilibria, but an asymmetric equilibrium in which one firm concentrates on trying to acquire only a few patents in an attempt to hold up the other firm can be ruled out. Suppose, for instance, $y_1 > 0$ and $y_i = 0$ for all $i > 1$. Then the optimal reply by firm A is to maximize

$$\pi_A = \frac{\gamma^n M}{2}\left(1 + \frac{x_1}{x_1 + y_1}\prod_{i=2}^{i=n} p_i(x_i, 0) - \frac{y_1}{x_1 + y_1}\prod_{i=2}^{i=n}(1 - p_i(x_i, 0))\right) + \sum_{i=1}^{n}\gamma p_i(x_i, y_i)R - \Sigma_i x_i.$$

Given that $p_i(x_i, 0) = 1/2$ for $x_i = 0$ but $p_i(x_i, 0) = 1$ for any $x_i > 0$, $x_i = 0$ is suboptimal. Firm A prefers to choose a small positive amount of effort that is feasible, say ϵ. But for any choice of small positive ϵ, the choice of $y_i = 0$ for $i = 2, 3, \ldots, n$ is suboptimal for firm B. Similar reasoning applies for cases in which player B chooses zero effort for any number $m \in \{1, \ldots, n-1\}$ of patent competitions.

However, patterns change for sufficiently high thresholds. We focus on the equilibrium for $k = 0$, as this is probably the most interesting case. A complete characterization of all equilibria that emerge for $\theta > x^*$

[12] In this case a mixed-strategy equilibrium exists. It is characterized by firm B choosing

$$y_i^* = \left(\frac{n-1}{n}\right)^n \frac{M}{2n}\frac{1}{n-1}$$

and firm A choosing $\mathbf{x}^* = (y_i^*/q^*, \ldots, y_i^*/q^*)$ with probability $q^* = 1/(n-1)$, and $\mathbf{x}^* = (0, \ldots, 0)$ with probability $1 - q^*$. Firm A's expected payoff is $\pi_A^* = M/2$, and firm B's expected payoff is $\pi_B = M/2(1 - (2(n-1)^{n-1})/n^n)$ in this equilibrium. This has been shown in a previous version of this paper (Clark and Konrad 2006).

is not analytically tractable. However, we can identify some equilibria that are qualitatively different from the symmetric equilibrium with positive efforts, as in Proposition 1. If the threshold θ is excessively high, no research will be carried out. The next proposition states a sufficiently high limit.

Proposition 4. *If $\theta > (\gamma^n M/n) + \gamma R$, then $\mathbf{x} = \mathbf{y} = \mathbf{0}$ is the unique equilibrium.*

Proof. We show that $\mathbf{x} = 0$ is a strictly dominant strategy. Let $\mathbf{y}$ be arbitrary. Any choice $x_i \in (0, \theta)$ is dominated by $x_i = 0$. Note that $\pi_A(\mathbf{x}, \mathbf{y}) \leq \pi_A(\mathbf{x}, \mathbf{y} = \mathbf{0})$. Moreover, $\pi_A(\mathbf{x}, \mathbf{y} = \mathbf{0}) = \gamma^n M + n\gamma R - n\sum_{i=1}^n x_i \leq \gamma^n M + n\gamma R - n\theta < 0$. □

Intuitively, if the rent that is associated with investing in patents as a monopolist does not justify the high innovation effort, then even a firm that obtains the maximum possible benefit of innovation would not like to invest. Accordingly, high innovation thresholds will preclude the innovation. For intermediate values of θ, other more interesting equilibria can emerge. In these cases the rent that can be gained from innovative activity is sufficiently large such that a firm would invest if the other firm does not invest in R&D. However, the rent is sufficiently small to rule out the type of symmetric effort equilibrium in Propositions 1, 2, and 3.

Proposition 5. *Let $k = 0$ and suppose $\theta \in [\gamma R + (\gamma^n M/2n) - \delta, \gamma R + \gamma^n M/(2n))$ for small but positive δ. Let n be an even number with $n \geq 2$. Then a set of noncooperative coordination equilibria exists in which firms noncooperatively partition the set of patents and each firm expends effort only on an exclusive subset of patents. One such equilibrium is characterized by $x_i = y_{(n/2)+i} = \theta$ and $x_{(n/2)+i} = y_i = 0$ for all $i = 1, 2, \ldots, (n/2)$.*

Proof. Suppose that $\mathbf{y} = (0, 0, \ldots, 0, \theta, \theta, \ldots, \theta)$. Consider the optimal reply by firm A. By a variant of Lemma 1, the optimal reply is uniform within the sets of patents $i = 1, \ldots, (n/2)$, and $i = (n/2) + 1, \ldots, n$. Denote these optimal effort levels as x_I and x_{II}. This turns the consolidated payoff function of firm A into

$$\pi_A(x_I, x_{II}) = \begin{cases} -\dfrac{n}{2}x_I - \dfrac{n}{2}x_{II} \\ \qquad \text{if } x_I < \theta \text{ and } x_{II} < \theta, \\ -\dfrac{n}{2}x_I + \dfrac{n}{2}\left(\dfrac{x_{II}}{\theta + x_{II}}\gamma R - x_{II}\right) \\ \qquad \text{if } x_I < \theta \text{ and } x_{II} \geq \theta, \\ \gamma^n \dfrac{M}{2} + \dfrac{n}{2}\gamma R - \dfrac{n}{2}x_I - \dfrac{n}{2}x_{II} \\ \qquad \text{if } x_I \geq \theta \text{ and } x_{II} < \theta, \\ \gamma^n \dfrac{M}{2} + \dfrac{n}{2}\gamma R + H - \dfrac{n}{2}x_I - \dfrac{n}{2}x_{II} \\ \qquad \text{if } x_I \geq \theta \text{ and } x_{II} \geq \theta \end{cases} \tag{21}$$

with

$$H = \gamma^n \frac{M}{2}\left(\frac{x_{II}}{\theta + x_{II}}\right)^{n/2} + \frac{n}{2}\gamma R \frac{x_{II}}{\theta + x_{II}}.$$

Inspection of this payoff can reduce the set of possible optimal replies (x_I, x_{II}). Any $x_I \in (0, \theta)$ or $x_{II} \in (0, \theta)$ is dominated by $x_I = 0$ and $x_{II} = 0$, respectively. Further, $\pi_A = 0$ is a lower bound for the maximum, as $\pi_A(0, 0) = 0$. Any $x_I > \theta$ is dominated by $x_I = \theta$, as $\partial\pi_A(x_I, x_{II})/\partial x_I = -(n/2)$ for all $x_I > \theta$. Further, $\pi_A(\theta, x_{II}) - \pi_A(0, x_{II}) \geq \gamma^n(M/2) + (n/2)\gamma R - (n/2)\theta - (n/2)x_{II} + (n/2)x_{II} = \gamma^n(M/2) + (n/2)\gamma R - (n/2)\theta$, which is positive as long as $\theta < \gamma^n(M/n) + \gamma R$ (the upper bound in Proposition 4). This shows that $x_I = \theta$ is optimal. It remains to show that $x_{II} = 0$ is optimal, given $x_I = \theta$. Note that $\Delta \equiv \pi_A(\theta, x_{II}) - \pi_A(\theta, 0) = (\gamma^n(M/2)(x_{II}/\theta + x_{II}))^{n/2} + (n/2)\gamma R(x_{II}/\theta + x_{II})) - (n/2)x_{II}$ for $x_{II} \geq \theta$. Note that the highest reasonable amount of $(n/2)x_{II}$ is smaller than $\gamma^n(M/2) + (n/2)\gamma R$, as this is the valuation of winning all patents $(n/2) + 1, \ldots, n$ with probability 1, and any finite θ makes A win the patents $(n/2) + 1, \ldots, n$ with a probability of less than 1. Accordingly, the maximum win probability that could occur for any reasonable choice of x_{II} is

$$q_{II}^* \leq \lim_{\delta \to 0} \frac{(\gamma^n(M/n) + \gamma R)}{(\gamma R + (\gamma^n M/2n) - \delta) + (\gamma^n(M/n) + \gamma R)} \leq \frac{2}{3}.$$

Note further that the minimum effort level for $x_{II} \in [\theta, \infty)$ is $x_{II} = \theta$. Suppose that the firm chooses this minimum effort level but has the impact q_{II}^*, which is clearly a generous approximation for any $\delta \to 0$. Hence, $\lim_{\delta \to 0} \Delta < (\gamma^n(M/2)(\frac{2}{3})^{n/2} + (n/2)\gamma R\frac{2}{3}) - n/2(\gamma R + \gamma^n M/2n) = \gamma^n M/2((\frac{2}{3})^{n/2} - \frac{1}{2}) - (n/2)\gamma R\frac{1}{3} < 0$. This holds for all $M + R \geq 0$, for $n/2 \geq 2$. For $n = 2$, a separate proof is straightforward: $\Delta_{(n=2)} = (\gamma^2 M/2 + \gamma R)(x_{II}/\theta + x_{II}) - x_{II}$, and $\theta = \gamma R + (\gamma^2 M/4) - \delta$. This is a standard symmetric rent-seeking contest for a prize of size $(\gamma^2 M/2 + \gamma R)$. The optimal reply to the opponent's effort of θ is smaller (larger) than θ if θ is larger (smaller) than $\frac{1}{4}(\gamma^2 M/2 + \gamma R)$. This amount is smaller than $\gamma R + (\gamma^2 M/4) - \delta = \theta$ for sufficiently small positive δ, and this concludes the proof. □

Proposition 5 suggests that high thresholds in the innovation effort may make outcomes in which firms duplicate research effort unprofitable. In this case firms may coordinate their efforts in the noncooperative equilibrium and avoid duplication of effort.[13] Note that this outcome is fully noncooperative and

[13] Coordination in noncooperative equilibrium always begs the question of how coordination can be achieved. One potential instrument for overcoming the coordination problem in this equilibrium may be voluntary standard-setting organizations. We are grateful to an associate editor of this journal for alerting us to this.

does not require collusion. In these types of equilibria, firms innovate for the independent value of the patent and to secure some patents as bargaining chips in future negotiations.

Note also that the partition of patents in this noncooperative equilibrium need not be symmetric. For instance, let $R = 0$ and $\gamma = 1$. Further, let the threshold efforts for the patents be $\theta_1 = \cdots = \theta_{n-1} = \theta$ and $\theta_n = (n-1)\theta$. Then $\mathbf{x} = (0, 0, \ldots, 0, \theta_n)$ and $\mathbf{y} = (\theta, \theta, \theta, \ldots, 0)$ are mutually optimal replies if $\theta \in [M/(n-1) - \delta, M/(n-1)]$. This can be shown by arguments fully parallel to the ones used in the proof of Proposition 5.

6. Inventing Around

The analysis in §§2–5 maps the problem posed by Heller and Eisenberg (1998) in its pure and extreme form, where all patents are perfect complements: each of them is needed for the product and does not have a substitute. Sometimes patents can be "invented around" in the sense that one particular component can be replaced by another, with or without a loss in quality of the product or a change in production cost.

The option to invent around changes the outside options when the firms negotiate and trade their patent rights with each other. The insights in previous sections remain valid in this case, but further insights can be gained. Staying as close as possible to our previous framework, we assume that the invent-around decisions are made during the negotiations, which possibly yields a patent trade, and that each patent that has been granted at Stage 1 can be invented around for the same fixed cost S.

First note that the option to invent around is useless if $S > M/2$. No firm would ever invent around a patent in this case, as $M/2$ is the price to be paid for the patent right transfer in the absence of invent arounds. Accordingly, invent arounds do not change our results at all if they are sufficiently costly.

If S is lower, then inventing around becomes a meaningful alternative to purchasing the missing patent rights from the other firm. To illustrate the implications, assume that $S \in [M/4, M/2]$. Suppose first that $n_A = n-1$ and $n_B = 1$. Firm A holds all but one patent, which was awarded to firm B. In this case, if bargaining fails, the reservation profits are $M - S$ for firm A and 0 for firm B: firm A can invent around the one missing patent, whereas for firm B it will be too expensive to invent around all $n-1$ missing patents. These payoffs denote the reservation payoffs in the bargaining that takes place between firms A and B in Stage 2. The total gain in the two firms' payoff from efficient negotiations is equal to S, the cost saving they have from using the existing patents instead of inventing around one of them. In the symmetric Nash bargaining solution the firms will split this gain equally. Hence, firm B receives $S/2$ in compensation for transferring the patent right in the one patent it holds.

Suppose now that $n_A = n - j$ for $j \in \{2, \ldots, (n+1)/2\}$, and $n_B = n - n_A$. In these cases it is more costly for firm A to invent around all missing patents. The reservation payoffs from a breakdown of negotiations becomes 0 for both firms, as in §3. Accordingly, the gain from efficient bargaining is equal to M, and symmetric Nash bargaining will yield a transfer of all patents to one firm for a price equal to $M/2$ in all these cases.

These equilibrium outcomes in Stage 2 can be used to consider the payoff functions of players in Stage 1, where we concentrate on the case $\gamma = 1, k = 0, \theta = 0$:

$$\begin{aligned}\pi_A = {} & \frac{M}{2} + \prod_{i=1}^{i=n} p_i(x_i, y_i)\frac{M}{2} - \prod_{i=1}^{i=n}(1 - p_i(x_i, y_i))\frac{M}{2} \\ & + \sum_{i=1}^{n} p_i(x_i, y_i)R - \Sigma_i x_i \\ & + \sum_{j=1}^{n}\prod_{\substack{i=1\\ i\neq j}}^{i=n} p_i(x_i, y_i)(1 - p_j(x_j, y_j))\frac{M-S}{2} \\ & - \sum_{j=1}^{n}\prod_{\substack{i=1\\ i\neq j}}^{i=n}(1 - p_i(x_i, y_i))p_j(x_j, y_j)\frac{M-S}{2}. \end{aligned} \tag{22}$$

This payoff is the same as in Equation (7) in the first line in (22) but has to be adjusted for the higher payoff that emerges to firm A in all cases in which he is awarded $n-1$ patents and firm B holds only one patent, and for the lower payoff that emerges for player A in all cases in which he is awarded only one patent and player B holds all other patents. These differences are added/subtracted in the two summation terms in the second line in (22). The problem is still symmetric between firms, and all patents are equally attractive in the patent race at Stage 1. Anticipating again that firms expend the same effort along all patents, (22) can be rewritten as

$$\begin{aligned}\pi_A = {} & \frac{M}{2} + p(x, y)^n\frac{M}{2} - (1 - p(x, y))^n\frac{M}{2} + np(x, y)R - nx \\ & + n(p(x, y))^{n-1}(1 - p(x, y))\frac{M-S}{2} \\ & - n(1 - p(x, y))^{n-1}p(x, y)\frac{M-S}{2}. \end{aligned} \tag{23}$$

For $n > 2$, the first-order conditions in the symmetric equilibrium show that the option to invent around increases the innovation incentives. Instead of (4) with $\gamma = 1$, they become

$$x^0 = \frac{M}{2^{n+1}} + \frac{R}{4} + \frac{M-S}{2^{n+1}}(n-2) = x^* + \frac{M-S}{2^{n+1}}(n-2). \tag{24}$$

Intuitively, the option to invent around weakens complementarity. Complementarity and the strong hold-up it can generate emerges only in situations in which the firm with the larger number of patents wins less than $n-1$ patents. The marginal benefit of winning one instead of no patents is reduced from $M/2$ to $S/2$, but the marginal benefit of winning $n-1$ instead of $n-2$ patents is increased from 0 to $(M-S)/2$.

This example illustrates how the option to invent around patents can weaken the otherwise strong implications of patent complementarity. First, firms' payoffs become a smoother function of the number of patents that are awarded to a firm; an additional patent, other than the first or the nth, may be valuable and increase the value of the firm. In addition to the independent value that each patent may have (R in the benchmark model), additional patents may reduce the hold-up problem a firm faces regarding the multicomponent good. Second, invent-around options may increase rather than decrease innovation incentives inherent in a multicomponent product, because these options reduce the complementarity problem. The potential to invent around a patent acts as insurance, making it more likely that a firm can monopolize the rent, and hence the incentive to innovate is increased.[14]

Here we concentrated on the case with $k=0$. The option to invent around also has interesting implications for $k_A>0$ and $k_B=0$. Particularly if θ is not negligible, it need not be optimal for firm B to spread effort evenly across all remaining patents. Similarly, if $\theta=0$ but the cost of inventing around is low only for a subset of the remaining patents, firm B may want to focus on one or a few patents that are difficult to invent around, and this may also cause focused and asymmetric equilibrium outcomes.

7. Discussion and Conclusions

Complementarity of multiple patents has been identified recently—by academics and industry practitioners—as a potential problem, particularly in biotechnology and other nascent industries based on complex technologies. We have considered the incentives for cumulative R&D effort if firms need several complementary patent rights for producing multicomponent products. Complementarity weakens the incentives to invest in R&D effort. Intuitively, when many complementary patents are needed to produce a particular multicomponent good, a firm will very likely fail to obtain all patents even if it invests heavily in each of the single patent contests. At the same time, holding a single patent secures the firm veto power, and its payoff is therefore the same whether it holds one, two, or even all but one of the patents. This makes it less worthwhile to spend much effort trying to win all the simultaneous contests. This effect is strengthened if there is technological uncertainty regarding whether each single patented innovation is technically feasible and is weakened if patents have a commercial value apart from the complementary use in the multicomponent application.

When some firms already hold some patents in their portfolios and others do not, this yields some secure payoff to the leading firms. Although this is good news for owners of firms that already hold large stocks of patents, such stocks also yield a disincentive for these leading firms to invest in ongoing patent contests, unless only one patent remains unwon. With a large number of further patents, the leading firms are discouraged from participating in these contests. The reason for this discouragement effect is very different from Arrow's well-known replacement effect by which a new, superior patent invalidates an incumbent firm's existing one. Instead, the strategic game becomes asymmetric because the firm that has a nonempty portfolio still needs to win all further patents to make the multicomponent product as a monopolist, whereas the competing firm needs to win only one of these further patents to secure some of the monopoly profit.

Although we do consider technological uncertainty, the patent race is motivated by rent seeking among firms at the equilibrium. A narrow interpretation of our results could then suggest that a reduction in patenting effort is beneficial. However, our results should not be interpreted in this sense. The analysis reveals important R&D research disincentives, related to the number of complementary patents, the relative importance of the multicomponent application compared to the independent values of patents, and the general uncertainty about the technological feasibility of innovations and patent protection. In a world with multicomponent products needing many patents, there may be considerable incentive to pursue R&D to acquire a portfolio of patents that can be used as trading chips by firms who face a hold-up, and by other firms to generate a hold-up. If innovation investment is "lumpy," such that a threshold level has to be crossed to be in the running for a patent, we find equilibria in which firms target a different set of patents, avoiding costly duplication. Thornhill (2006) suggests that, empirically, the threshold might be quite low in stable industries with a lower pace of technological change. In this case, our model demonstrates the possibility of symmetric equilibria in which each firm competes for all patents as long as they have sufficient individual worth. In more dynamic industries, the threshold level of investment may be higher, and equilibrium

[14] We are grateful to Kalle Moene for pointing this out to us.

behavior reflects firms' desire to avoid duplication of large costs.

We have also considered firms' options to "invent around" a given patent at some cost. This option weakens the otherwise strong technological complementarity of the set of patents that is needed for the multicomponent product and can increase the equilibrium R&D effort. The option to invent around hence makes the acquisition of patents more desirable in the context of strict complementarity and reduces the hold-up problem that underlies the original observation by Heller and Eisenberg (1998). From a practical perspective, the invent-around option may also explain why the complementarity problem is less pronounced in most industries than is suggested in theory.

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Appendix

Optimality of Uniform Effort. We prove the following lemma:

Lemma 1. *Let* $\pi_A(\mathbf{x},\mathbf{y})$ *be given by* (7) *or by* (14). *Let* $\mathbf{y}=(y_1,\ldots,y_n)$ *with* $y_i=y_j\equiv\theta>0$ *and* $\mathbf{x}=(x_1,\ldots,x_n)$, *with* $x_i\neq x_j$ *for some* $i,j\in\{1,\ldots,n\}$. *Define the vector* $\check{\mathbf{x}}$ *such that* $\check{x}_k=x_k$ *for all* $k\notin\{i,j\}$, *and* $\check{x}_i=\check{x}_j=(x_i+x_j)/2$. *Then* $\pi_A(\check{\mathbf{x}},\mathbf{y})\geq\pi_A(\mathbf{x},\mathbf{y})$. *Let* $\pi_B(\mathbf{x},\mathbf{y})$ *be given by* (15). *Let* $\mathbf{x}=(x_1,\ldots,x_n)$ *with* $x_i=x_j\equiv\theta>0$ *and* $\mathbf{y}=(y_1,\ldots,y_n)$, *with* $y_i\neq y_j$ *for some* $i,j\in\{1,\ldots,n\}$. *Define the vector* $\check{\mathbf{y}}$ *such that* $\check{y}_k=y_k$ *for all* $k\notin\{i,j\}$, *and* $\check{y}_i=\check{y}_j=(y_i+y_j)/2$. *Then* $\pi_B(\mathbf{x},\check{\mathbf{y}})\geq\pi_B(\mathbf{x},\mathbf{y})$.

Proof. Consider first (7):

$$\begin{aligned}\pi_A(\check{\mathbf{x}},\mathbf{y})-\pi_A(\mathbf{x},\mathbf{y})&\\=\bigg[p_i\Big(\frac{x_i+x_j}{2},\theta\Big)p_j\Big(\frac{x_i+x_j}{2},\theta\Big)-p_i(x_i,\theta)p_j(x_j,\theta)\bigg]\Psi&\\-\bigg[\Big(1-p_i\Big(\frac{x_i+x_j}{2},\theta\Big)\Big)\Big(1-p_j\Big(\frac{x_i+x_j}{2},\theta\Big)\Big)&\\-(1-p_i(x_i,\theta))(1-p_j(x_j,\theta))\bigg]\Phi&\\+\bigg[p_i\Big(\frac{x_i+x_j}{2},\theta\Big)-p_i(x_i,\theta)+p_j\Big(\frac{x_i+x_j}{2},\theta\Big)-p_j(x_j,\theta)\bigg]\gamma R,&\end{aligned}\tag{25}$$

with $\Psi\equiv(\gamma^nM/2)\Pi_{k=1,k\neq i,j}^{n}p_k(x_k,y_k)\geq0$ and independent of x_i or x_j, and $\Phi\equiv(\gamma^nM/2)\Pi_{k=1,k\neq i,j}^{n}(1-p_k(x_k,y_k))\geq0$ and independent of x_i or x_j. Each of these three summands is nonnegative. First,

$$\begin{aligned}p_i\Big(\frac{x_i+x_j}{2},\theta\Big)p_j\Big(\frac{x_i+x_j}{2},\theta\Big)-p_i(x_i,\theta)p_j(x_j,\theta)&\\=\frac{(x_i+x_j)/2}{(x_i+x_j)/2+\theta}\frac{(x_i+x_j)/2}{(x_i+x_j)/2+\theta}-\frac{x_i}{x_i+\theta}\frac{x_j}{x_j+\theta}&\\=\frac{(x_i-x_j)^2(x_i+x_j+\theta)\theta}{(x_i+x_j+2\theta)^2(x_i+\theta)(x_j+\theta)}\geq0;&\end{aligned}\tag{26}$$

second,

$$\begin{aligned}-\bigg[\Big(1-p_i\Big(\frac{x_i+x_j}{2},\theta\Big)\Big)\Big(1-p_j\Big(\frac{x_i+x_j}{2},\theta\Big)\Big)&\\-(1-p_i(x_i,\theta))(1-p_j(x_j,\theta))\bigg]&\\=-\frac{\theta}{(x_i+x_j)/2+\theta}\frac{\theta}{(x_i+x_j)/2+\theta}+\frac{\theta}{x_i+\theta}\frac{\theta}{x_j+\theta}&\\=\frac{(x_i-x_j)^2\theta^2}{(x_i+x_j+2\theta)^2(x_i+\theta)(x_j+\theta)}\geq0;&\end{aligned}\tag{27}$$

and third,

$$\begin{aligned}p_i\Big(\frac{x_i+x_j}{2},\theta\Big)-p_i(x_i,\theta)+p_j\Big(\frac{x_i+x_j}{2},\theta\Big)-p_j(x_j,\theta)&\\=2\frac{(x_i+x_j)/2}{(x_i+x_j)/2+\theta}-\frac{x_i}{x_i+\theta}-\frac{x_j}{x_j+\theta}&\\=\frac{(x_i-x_j)^2\theta}{(x_i+x_j+2\theta)(x_i+\theta)(x_j+\theta)}\geq0.&\end{aligned}\tag{28}$$

For $\pi_B(\mathbf{x},\mathbf{y})$ in Proposition 1 the same argument holds, as A and B are fully symmetric. For $\pi_A(\mathbf{x},\mathbf{y})$ as in (14), we find that

$$\begin{aligned}\pi_A(\check{\mathbf{x}},\mathbf{y})-\pi_A(\mathbf{x},\mathbf{y})&\\=\bigg[p_i\Big(\frac{x_i+x_j}{2},\theta\Big)p_j\Big(\frac{x_i+x_j}{2},\theta\Big)-p_i(x_i,\theta)p_j(x_j,\theta)\bigg]\Psi&\\+\bigg[p_i\Big(\frac{x_i+x_j}{2},\theta\Big)-p_i(x_i,\theta)&\\+p_j\Big(\frac{x_i+x_j}{2},\theta\Big)-p_j(x_j,\theta)\bigg]\gamma R.&\end{aligned}\tag{29}$$

Hence, this difference is nonnegative by (26) and (28).

Finally, the case for (15) can be made along analogous lines:

$$\begin{aligned}\pi_B(\mathbf{x},\check{\mathbf{y}})-\pi_A(\mathbf{x},\mathbf{y})&\\=-\bigg[p_i\Big(\theta,\frac{y_i+y_j}{2}\Big)p_j\Big(\theta,\frac{y_i+y_j}{2}\Big)-p_i(\theta,y_i)p_j(\theta,y_j)\bigg]\Xi&\\+\bigg[1-p_i\Big(\theta,\frac{y_i+y_j}{2}\Big)-(1-p_i(\theta,y_i))&\\+\Big(1-p_j\Big(\theta,\frac{y_i+y_j}{2}\Big)\Big)-(1-p_j(\theta,y_j))\bigg]\gamma R&\end{aligned}\tag{30}$$

with $\Xi\equiv(\gamma^nM/2)\Pi_{k=1,k\neq i,j}^{n}p_k(\theta,y_k)\geq0$. Using $p_i(\theta,v_i)=(1-p_i(v_i,\theta))$ and $(1-p_i(v_i,\theta))=p_i(v_i,\theta)$, we apply (27) and (28) to show that the terms in lines 2 and 3 of (30) are nonnegative. □

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