MERGER PROFITABILITY IN INDUSTRIES WITH BRAND PORTFOLIOS AND LOYAL CUSTOMERS

KAI A. KONRAD*

We study the equilibrium effects of mergers between firms with brand portfolios and brand loyal customers for pricing and profitability. We find that the “merger paradox” (Salant, Switzer and Reynolds 1983) is absent in these markets. The acquisition of brand portfolios can be profit enhancing for the merging firms and payoff neutral for the firms not involved in the merger. This may explain the emergence of brand conglomerates such as Richemont, PPR or LVMH.

JEL Classification: D43, L22, M31
Keywords: brand portfolios, merger profitability, customer loyalty

I. INTRODUCTION

Horizontal mergers of firms in markets with well-known brand names is a frequent phenomenon. The brand portfolio of an acquisition target is often an important co-determinant of the value of the acquired firm\(^1\), and the acquired firm’s portfolio of brands is often continued and promoted

\(^1\) According to Bahadir, Bharadwaj and Srivastava (2008) the value of brands owned by the target firm is substantial, and for some firms they report that the brand portfolio value accounted for about one half of the firm value. In their theory they focus on marketing synergies and economies of scale.
by the acquiring company. Some important examples can be seen in the car industry, luxury consumer products, and fashion industry. We study the merger profitability in markets which are characterized by such firms with multiple brands. We assume that customers can either be price sensitive, or may be loyal to one or the other brand, purchasing a product of this brand if and only if the price of the product is not higher than some reservation price. Firms may own several brands which constitute their brand portfolio. They may make pricing decisions on each of its brand in their portfolio. We consider the profitability of mergers and acquisitions between firms with multiple brands. We ask how the profitability of merger depends on the composition of the brand portfolio, and how the merger affects bystanding firms which are not involved in the merger.

The analysis of motives for mergers and acquisitions and the implications of such merger for profitability and welfare has been a field of very active research for the last 25 years. The formal study of the equilibrium effects on profitability of merger has an important starting point in the merger paradox that was derived by Salant, Switzer and Reynolds (1983). Their analysis of mergers in a symmetric Cournot market with constant marginal cost showed that such a merger is typically unprofitable for the firms that merge, whereas bystanding firms benefit from the increase in concentration. A complementary paper by

2 Bahadir, Bharadwaj and Srivastava (2008) discuss why this policy is more common if the acquiring firm has a diversified brand portfolio (e.g., GM), compared to a firm with a single or very few strong brands (e.g., GM) that may decide to disconnect some of the target firms brands.

3 Several car producers have acquired a whole number of other brands. Volkswagen, for instance, absorbed firms such as Audi, Skoda, Seat, some high-status labels such as Bugatti, Lamborghini and Bentley, and Porsche in 2009. Similarly, BMW absorbed Mini and Rolls-Royce and tried to integrate Rover, and Ford acquired Jaguar, Land Rover and Volvo, plus shares in Mazda and Aston Martin.

4 For instance, the company LVMH was born from a merger of Moët Hennessy and Louis Vuitton S.A. Both firms own very strong brands in the segment of luxury consumer products, where LVMH itself is partially owned by the haute couture fashion retailer Christian Dior (see, e.g., http://www.lvmh.com/fonctionalite/fq_faq_histo.asp) The conglomerate PPR, formerly known as Pinault-Printemps-Redoute, owns Gucci, which, itself, owns strong brands such as Yves Saint Laurent, Sergio Rossi, Boucheron, Bottega Veneta, Bédat & Co, Alexander McQueen, Stella McCartney, Balenciaga (See http://www.ppr.com/front_sectionId-183_Changelang-en.html).

5 Their basic argument is intuitive and robust. If, for example, three identical firms A, B and C compete in a Cournot market, each of the firms makes a profit equal to 1/3 of the oligopoly profit that emerges in the market with three active firms. If firms B and C merge into B&C, from a strategic point of view this leads to a duopoly with two symmetric firms. The whole industry profit in this market increases from that of an oligopoly with three firms to the duopoly profit. But the
Deneckere and Davidson (1985) who consider Bertrand markets with differentiated products is the starting point of a long series of studies that describe conditions for which the merger paradox is moderated. A recent (non-exhaustive) survey on the merger paradox is by Huck, Konrad and Müller (2008). However, a milder version of the merger paradox remains even for many of these studies, including the case of Bertrand competition, as the bystanding firms would often gain more from the merger than the merging firms, essentially leading to a situation in which all firms like mergers, but prefer to let other firms merge. In our analysis of merger between firms with multiple brands and brand-loyal customers, the merger is either profitable for the merging firms or does not affect their profits. The profits of bystanding firms are unaffected. We build on a stock of results from the theory of price competition between firms who have groups of loyal customers and who also compete for groups of customers who are price sensitive and not loyal to only one or the other brand. This type of competition theory originated with Varian (1980) and developed rapidly, with important contributions by Narasimhan (1988), Baye, Kovenock and de Vries (1992) and many others. In Bertrand competition with loyal customer groups, when making pricing decisions firms must decide whether to choose a high price, by which they are likely to lose all non-loyal customers to other firms and most likely sell to their loyal customers only, or whether they would also like to compete for...
the price-sensitive customers. In the latter case, they have to lower their prices, implying that they also sell to their loyal customers at these lower prices. One important property of this type of competition is that it establishes a situation in which many firms can sustain high prices, with only very few firms being engaged in price competition.\(^8\)

Brands may differ in the size of their loyal customer groups, with “weak” brands having few and “strong” brands having many loyal customers. We show that the composition of firms’ brand portfolios matters. The relative size of loyal customer groups in the weaker brands is a key element for the question whether a merger among firms with brands with loyal customers is profitable or not, and whether such a merger harms or benefits other non-merging firms in this industry. We find that the acquisition of firms with one or several brands may but need not change the distribution of prices in the Bertrand equilibrium. The relative size of loyal customers of the weakest brands (their “strength”) in the acquiring firm and in the firm acquired matters. A merger that brings together a set of very strong brands does not affect the pricing equilibrium. There may be possible scale economies and a possible change in the strength of brands due to the movement of ownership of the brand from one firm to another, which may be profit relevant. We remove such effects from the picture and focus on the pure effects of changes in equilibrium pricing. A merger that brings together firms with the weakest brands in their portfolio can change the equilibrium pricing and typically has a positive effect on profitability for the firms who engage in the merger, and no profitability effects for all other firms.

Empirically, the role of heterogeneity of customers with some customer groups being loyal to specific brands and other customers being sensitive only to prices, is important at least in some markets. Brands play a prominent role in the car market. Many of the large car companies support and market a whole set of brands. GM and Volkswagen are prominent examples.\(^9\) Similarly, a series of mergers and acquisitions led

---

\(^8\) Lal and Villas-Boas (1998) broaden the picture to allow for vertical supply structures with different combinations of customer loyalty (see also Shaffer and Zhang 2002 and Srinivasan, Pauwels, Hanssens and Dekimpe 2004).

\(^9\) It is important to note that brands are not just horizontally differentiated products. Volkswagen and its subsidiary, Audi, produce a whole set of models and many of these models correspond most closely with each other. From a purely technical point of view, some of their
to Richemont, a company that owns, for instance, Cartier, Van Cleef & Arpels, Piaget, Vacheron Constantin, A. Lange & Söhne, Jaeger-LeCoultre, Officine Panerai, International Watch Co, and Baume et Mercier, which are all high-end producers of jewelry and/or wrist watches, and a number of further brands, such as Montblanc or Alfred Dunhill. Similar to the car industry example, it can hardly be argued that the different watches produced by these subsidiaries are differentiated horizontally or vertically along purely functional or quality dimensions. The main difference between the different sets of watches produced is, seemingly, their brand name. These examples suggest that what firms acquired in these processes was not mainly aimed at owning a balanced portfolio of differentiated products, but that the acquisition of brands was a key element of these acquisitions of firms, as acquiring a brand essentially involved the acquisition of a set of loyal customers.

We proceed as follows. In section 2 we formally review some of the results in the literature which we use for analysing merger of multi-brand firms and analyse merger between single-brand firms. We then turn to the main contribution in this paper and analyse merger between multi-brand firms. Section 3 offers conclusions.

II. THE MERGER ANALYSIS

We consider the following analytical framework. There is a set $S$ of brand names $i$, with $i = 1, 2, \ldots s$. In the benchmark case which is our point of departure, the number of firms $1, \ldots, s$ is the same as the number of brands and each firm $i$ produces the same good with the same constant unit cost normalized to zero for simplicity, owns one brand and sells its product using this brand name, chooses a price $p_i$ and offers to serve any demand at this price. The choices of prices are made simultaneously and independently by all firms. There is a large set $B$ of models are very close substitutes, or can even be seen as perfect substitutes, given that they are equipped with the same technology and are even partially produced using the same components. The key difference between these corresponding models is the difference in brand name, and this difference may be important due to brand loyalty. Rolls-Royce is another example. Rolls-Royce produced virtually the same car and sold it using two strong brands: Rolls-Royce and Bentley, the different radiator grills and cooler bodies being the main distinguishing elements.  

\footnotesize{10} See, e.g., http://www.richemont.com/our_businesses.html.
consumers which can be thought of as the unit interval with unit measure. Each consumer may buy exactly one unit from exactly one seller, or may not buy at all. The set of consumers is partitioned into \( s + 1 \) groups of size \( n_1, n_2, \ldots, n_s \) and \( m \). Consumers from the subset \( i \) are loyal to brand \( i \) for \( i = 1, \ldots, s \). They buy one unit of the good of brand \( i \) if the price \( p_i \) for this brand is not higher than their reservation price \( r \). We denote the share of consumer \( s \) which is loyal to brand \( i \) as \( n_i \), and we assume that brands are numbered according to their strength:

\[
0 \leq n_1 < n_2 < \ldots < n_s. \tag{1}
\]

Brand \( j \) is called weaker than brand \( j + 1 \), as it has a smaller group of loyal consumers. The weakest brand is brand 1, the strongest brand is brand \( s \). Strict inequality in (1) is assumed for simplicity, as this helps to eliminate non-generic multiple equilibria. Further, there is a group of size \( m \) of consumers who are not loyal to any of the brands. Hence, the share of non-loyal consumers is \( m > 0 \). Consumers who are not loyal purchase the good for the lowest price that is offered. This benchmark case describes the framework analyzed by Kocas and Kiyak (2006), which generalizes Narasimhan (1988) who considered two single-brand firms with \( n_1 \leq n_2 \), and Baye, Kovenock and deVries (1992) who considered more than two firms with one brand each, but equally strong brands.

We first compare this benchmark case with a situation which may result from a merger. In this alternative situation there is one multi-brand firm that owns the brands in the subset \( K \subset S \), with the number of elements in \( K \) denoted as \( \#K \), and \( \#K < s \) brands and a remaining set of firms which all own one brand.\(^{11}\) The multi-brand firm may, for instance, be the result of a merger, namely if the firms owning the set \( K = \{1, \ldots, \#K\} \subset S \) of brands merge and the resulting firm maintains all brands formerly owned by the single firms.\(^{12}\) For notational convenience, we assume that these brands are sorted by strength, with \( n_{i_k} < n_{i_{k+1}} \). The multi-brand firm then owns a portfolio of brands

\(^{11}\) A generalization from there to the situation with several multi-brand firms is straightforward and is discussed further below.

\(^{12}\) As discussed in the introduction, this is what often happened historically, for example, in the luxury consumer products industry or in the car industry.
1₁,…,(#K)ₖ. It therefore internalizes the effects of the choice of the price for one of its brands for sales in one of the other brands. This firm chooses a vector of prices \( pₖ \equiv (p₁, \ldots, p_{(vk)ₖ}) \) that maximizes this firm’s profits, taking the prices \( p_j \) chosen by all other single brand firms \( j \) as given. Similarly, these \( s-(#K) \) other firms with single brands \( j \not\in K \) choose their price \( p_j \) independently as in the benchmark case. Consumers who were loyal to one of the brands in the benchmark case are assumed to remain loyal to their old brand\(^{13}\), and customers without any brand loyalty in the benchmark case remain without brand loyalty.

Our focus is on the implications of merger in this framework and a comparison of firms’ equilibrium payoffs in the benchmark situation and in the situation with a multi-brand firm (i.e., after a merger). While we do not address the issue of endogeneity of mergers, the profitability of a merger for the merging firms and for the bystanding firms is an indication of the merger incentives if merger is endogenous.\(^{14}\)

We first recall the equilibrium solution for the benchmark case.

**Proposition 1** (Kocas and Kiyak 2006) An equilibrium is characterized by the following pricing strategies: all firms owning brands \( j = 3, \ldots, s \) choose \( p_j = r \). The firms owning brands 1 and 2 choose their prices as mixed strategies described by the following cumulative distribution functions:

\[
F_1(p₁) = 1 + \frac{n₂}{m} (1 - \frac{r}{p₁}) \quad \text{for} \quad p₁ \in \left[ \frac{n₂r}{n₂ + m}, r \right),
\]

\[
F_2(p₂) = 1 + \frac{n₁}{m} - \frac{(n₁ + m)n₂r}{(n₂ + m)mp₂} \quad \text{for} \quad p₂ \in \left[ \frac{n₂r}{n₂ + m}, r \right),
\]

\(^{13}\)It is not necessarily trivial to acquire a brand and still preserve customer loyalty for this brand (see Jaju, Joiner and Reddy 2006). The theoretical considerations by Baye, Crocker and Ju (1996) and Lommerud and Sørgard (1997) also show that the independence of brands in the process of a merger should not be taken for granted, as they essentially depart from this assumption.

\(^{14}\)There are many aspects of mergers other than the strategic aspects for market interaction. Among these are, for instance, possible economies of scale in production, marketing or advertising, cost of restructuring, information spillovers etc. These other aspects also matter for mergers and acquisitions, but when considering the strategic aspect of a merger for the interaction in the market that is at the heart of the merger paradox, it makes sense to remove these other aspects from the picture.
and \( F_1(p_i) = 0 \) for \( p_i \in [0, \frac{n_r}{n_m + n_r}] \) and \( F_i(p_i) = 1 \) for \( p_i \geq r \) for \( i = 1, 2 \). Firms’ payoffs are \( \pi_j = n r_j \) for all \( j = 2, \ldots, s \), and \( \pi_1 = \frac{n_r + \epsilon}{n_2 + \epsilon} n_2 r \).

A proof can be found in Kocas and Kiyak (2006). Some of the properties of the equilibrium can be explained in intuitive terms. Each firm chooses between two options: extracting a maximum of revenue from its loyal consumers by charging their reservation price, essentially leaving the competition for the non-loyal customers to others, or also competing for the non-loyal customers. In the latter case firm \( j \) chooses a price \( p_j < r \). Accordingly, competing for the set of non-loyal consumers has an opportunity cost: it reduces the margin that can be earned on the firm’s loyal consumers. This opportunity cost is higher for firms which have a stronger brand (i.e., a larger group of loyal customers). The firms with the weakest two brands have the lowest opportunity cost of lowering prices. This is a competitive advantage. In the equilibrium all strong brands stay out of this competition and simply extract maximally from their loyal consumers. Their competition leads to an equilibrium in mixed strategies. In the equilibrium they both randomize according to the cumulative distribution functions as in (2) and (3) that are the same as in the two-firm equilibrium analyzed by Narasimhan (1988). The lower bound of the common support of equilibrium prices is precisely the price at which the firm owning brand 2 (the second-weakest brand) is just indifferent between underbidding this price and winning all non-loyal customers or choosing its reservation price and serving only its own loyal customers.

Proposition 1 provides the point of departure for our analysis. The next proposition considers competition with multi-brand firms that result from a merger and compare the payoffs with the benchmark case.

**Proposition 2** Consider mergers that do not lead to a monopoly. (i) A merger that leads to a multi-brand firm with a set \( K \) of brands such that

15 An equilibrium in pure strategies for \( p_1 \) and \( p_2 \) can be ruled out: for each firm it is either superior to choose a price slightly smaller than a given price chosen by the competitor, or the price chosen by the competitor is so low that it is better not to compete for the non-loyal customers and to resort to the firm’s loyal consumers and charge their reservation price. But then the low price of the competitor is itself not an optimal reply.
\{1,2\} \subseteq K \text{ is profitable for the merging firms and does not change the equilibrium payoffs for all non-merging firms. (ii) If } \{1,2\} \not\subseteq K, \text{ then an equilibrium is characterized by the same pricing behavior as the ones described in Proposition 1, and the merger is neither profitable nor unprofitable.}

**Proof.** Consider part (i). Let \{1,2\} \subseteq K. Let \( h \) be the weakest brand for which \( h \not\in K \). We consider the following pricing strategies as a candidate for an equilibrium. First, \( p_j = r \) for all brands \( j \in (S \setminus \{1,h\}) \). Second, the multi-brand firm chooses \( p_1 \) according to

\[
F_1(p_1) = \begin{cases} 
0 & \text{for } p_1 \in [0, \frac{n_hr}{n_h + m}) \\
1 + \frac{n_h}{m} (1 - \frac{r}{p_1}) & \text{for } p_1 \in \left[\frac{n_hr}{n_h + m}, r\right) \\
1 & \text{for } p_1 \geq r.
\end{cases}
\]  

(4)

Third, the firm that owns brand \( h \) chooses \( p_h \) according to

\[
F_h(p_h) = \begin{cases} 
0 & \text{for } p_h \in [0, \frac{n_hr}{n_h + m}) \\
1 + \frac{n_h}{m} (\frac{n_1 + m)n_hr}{(n_h + m)mp_h} & \text{for } p_h \in \left[\frac{n_hr}{n_h + m}, r\right) \\
1 & \text{for } p_h \geq r
\end{cases}
\]  

(5)

Given these choices, firms’ payoffs are \( \pi_j \) for all single-brand firms including firm/brand \( h \). The multi-brand firm makes a profit equal to \( rn_j \) from each of its brands except from brand \( 1_k (=1) \), and the contribution to profit by brand \( 1 \) is \( \pi_1 = \frac{n_1 + m}{n_1 + m} n_hr \).

The merger is profitable for the merging firms if \( \frac{n_1 + m}{n_1 + m} n_hr > \frac{n_1 + m}{n_2 + m} n_hr \). This holds, as \( n_1 > n_2 \) holds as \( 2 \in K \). Note also that bystanding firms’ profits are unaffected by the merger.
We now show that these pricing strategies are mutually optimal replies. First, we confirm that $F_h$ maximizes $\pi_h$ given $F_i$ and $p_j = r$ for all other brand prices. Note that $\pi_h = (1 - F_i)p_jm + p_hn_h = (1 - (1 + \frac{n_h}{m})(1 - \frac{r}{p_j}))p_jm + p_hn_h = rn_h$ for any $p_h \in \left[\frac{nr}{n+m}, r\right]$, whereas $\pi_h = p_h(m + n_h) < n_hr$ for $p_h < \frac{nr}{n,m}$ and $\pi_h = 0$ for $p_h > r$. This proves the optimality of $F_h$ for the single-brand firm that owns brand $h$. Second, we confirm that $p_j = r$ maximizes $\pi_j$ for all other single brand firms which, by definition of $h$, have a larger group of loyal customers than brand $h$. Clearly, $p_j > r$ is dominated by $p_j = r$. Moreover, for $p_j < r$ the payoff is

$$\pi_j = (1 - F_i(p_j))(1 - F_h(p_j))p_jm + p_jn_j$$

$$\leq (1 - F_i(p_j))p_jm + p_jn_j$$

$$= (1 - (1 + \frac{n_h}{m})(1 - \frac{r}{p_j}))p_jm + p_jn_j$$

$$< rn_j$$

for all $p_i \leq r$. The latter inequality makes use of the property $n_j > n_h$.

Turn now to the optimality of pricing choices of the brands that constitute the merger group. Take $F_h$ and $p_j = r$ for $j \in S \setminus (K \cup \{h\})$ as given. The multi-brand firm chooses $p_K$. Let $p_{\min} = \min\{p_{i_1}, \ldots, p_{(i_K)\kappa}\}$ the smallest component of $p_K$. Then the multi-brand firm’s payoff is

$$\pi_K(p_K) = (1 - F_h(p_{\min}))p_{\min}m + \sum_{i_{\kappa} \in K} p_{i_{\kappa}}n_{i_{\kappa}},$$

if all $p_{i_{\kappa}} \leq r$ for $i_{\kappa} \in K$, and smaller if $p_{i_{\kappa}} > r$ for some $i_{\kappa} \in K$.

A necessary condition for this sum to be maximal for a given $p_{\min}$ is that $i_{\min} = 1_K (= 1)$, i.e., the weakest brand is assigned the lowest price. This can be confirmed as follows. The first term in (7) depends only on $p_{\min}$, but not on whether $i_{\min} = 1_K$ or not. If $i_{\min} = i_K = 1_K$ the second term in (7), $\sum_{i_{\kappa} \neq K}(p_{i_{\kappa}}n_{i_{\kappa}})$, can be increased by a joint adjustment of two prices: the price of brand $i_{\min}$ is replaced by the price previously assigned to brand $1_K$ and vice versa.
The necessary condition $i_{\text{min}} = 1_K$ can now be used to conclude that $p_{i_k} = r$ for all $i_k \neq 1_K$ is a necessary condition for (7) to be maximal. If $i_{\text{min}} = 1_K$, the payoff (7) can be increased monotonically by increasing all $p_{i_k}$ up to $p_{i_k} = r$ for all $i_k \neq 1_K$. This shows that the optimal reply is $p_{i_k} = r$ for all $i_k \neq 1_K$.

Given that $p_{i_k} = r$ for all $i_k \neq 1_K$, the optimality of $p_{i_k} \in [\frac{n_h \cdot r}{n_h + m}, r]$ can be shown by considering the multi-brand firm’s payoff as a function of $p_u$, given $p_{i_k} = r$ for all $i_k \neq 1_K$. This payoff is

$$\pi_K(p_{i_k}) = (1 - F_h(p_{i_k}))p_{i_k}m + p_{i_k}n_u + \sum_{i_k \in K \setminus \{1_K\}} rn_{i_k},$$  

(8)

The third term in (8) is independent of $p_{i_k}$. The sum of the first and second term in (8) is the same as if a single brand-firm owning brand $1_K(-1)$ would compete with the single-brand firm with owning brand $h$ only. Inserting $F_h$ from (5) it is straightforward to see that the sum of these terms is equal to $\frac{n_u \cdot r}{n_u + m} n_h r$ for all $p_{i_k} \in [\frac{n_h \cdot r}{n_h + m}, r]$, zero for $p_{i_k} < r$ and smaller than $\frac{n_h \cdot m}{n_u + m} n_h r$ for all $p_{i_k} \in [\frac{n_u \cdot r}{n_u + m}, r]$.

The case (iii) is relegated to the Appendix.

Part (i) is the more interesting part of Proposition 2. It shows that the formation of multi-brand firms can benefit the group of merging firms, provided that the weakest brands are inside this group. The benefit for the merging firms comes from the fact that the new multi-brand firm owns both brands that competed most fiercely in the benchmark case without merger. After the merger the multi-brand firm owning these brands can control the prices for all its brands and can prevent the brands from competing internally. This will not prevent other single-brand firms from competing for the non-loyal customers, and typically one of them will lower its price. However, as these non-acquired firms only have brands that are stronger than the weakest brands acquired and, hence, have higher opportunity costs in this competition, they will compete less aggressively, and this drives up the payoff earned on the weakest brand. In the benchmark case, the two weakest brands compete for the non-loyal customers. If both these brands are owned by the acquiring firm, the acquiring firm can order the second-weakest brand to charge the consumer reservation price $r$, rather than compete with brand 1 for non-
loyal customers. This relaxes competition and drives up the profits of the acquiring firm.

To illustrate the anti-competitive effect further with an example, let the three weakest brands with loyal consumer groups have size $n_1$, $n_2$ and $n_3$ and let the set of non-loyal consumers be of size $m$. In the benchmark case the equilibrium price for brand 3 is $p_3 = r$, whereas brands 1 and 2 compete choosing mixed strategies (2) and (3). In this competition the firms end up with profits $\pi_3 = rn_3$, $\pi_2 = rn_2$ and $\pi_1 = \frac{n_1 + m}{n_1 + n_2} nr$. If firm 1 acquires firm 2 (and, hence, brand 2), then firm 1 can control the pricing for brands 1 and 2 and can prevent brand 2 from competing against brand 1. In the new equilibrium, firm 1 still cannot simply choose to make $p_1$ slightly smaller than $r$ and to sell to all non-loyal customers, because this would draw firm 3 into the competition for the non-loyal customers. Firm 3 essentially assumes the former role of firm 2. The competition for the non-loyal customers will be between brand 1 and brand 3. The benefit for the acquiring firm emerges because firm/brand 3 is less aggressive than firm 2 in its pricing behavior, because firm/brand 3 has a higher opportunity cost of underbidding brand 1 than the opportunity cost of brand 2, because firm/brand 3 has a larger group of loyal customers than brand 2. As a result, the expected payoffs $\pi_2 = rn_2$ and $\pi_3 = rn_3$ remain unchanged, but the profit on brand 1 increases from $\frac{n_1 + m}{n_1 + n_2} nr$ to $\frac{n_1 + m}{n_1 + n_2} nr$.

The intuition for Proposition 2 carries over to a further acquisition by the multi-brand firm that enlarges its brand portfolio. Suppose for this purpose that $\{1, 2\} \subset K$, and $n_h = \min\{n_j \mid j \notin K\}$. Then it follows directly from Proposition 2 that any acquisition of a further single-brand firm other than the one that owns brand $h$ does not change the pricing equilibrium. The payoff of the multi-brand firm simply increases by $rn_j$ from acquiring such an additional single-brand firm. Such a further acquisition is not profitable. However, if the multi-brand firm acquires the firm owning brand $h$, then this changes the equilibrium. The equilibrium price for this brand in the new equilibrium becomes $p_h = r$, and the weakest brand that is not owned by the multi-brand firm takes over the former role of brand $h$. If this is brand $\hat{h}$, then $p_{\hat{h}}$ changes from $p_h = r$ to a mixed strategy described by a cumulative distribution function $F_{\hat{h}}$ as in (3) with $n_{\hat{h}}$ replacing $n_2$ in (2) and (3).
We can also discuss mergers starting from a case with several multi-brand firms. For this purpose let there be \( \nu > 2 \) firms, with each firm owning a (non-empty) portfolio of brands, with these portfolios denoted as sets \( K_1, \ldots, K_\nu \), such that \( \{K_1, \ldots, K_\nu\} \) is a partition of \( S \), and \( K_j = \{1, \ldots, (#K_j)\} \) for \( j \in \{1, \ldots, \nu\} \). Note that the case of single-brand firms is a special case. Further, let the weakest brands in the portfolios of each of the multi-brand firms be denoted as \( 1_1, \ldots, 1_\nu \), respectively, and let the numbering of firms be such that \( n_1 < \ldots < n_\nu \); i.e., the weakest brand in firm 1 is weaker than the weakest brand in firm 2 etc. up to firm \( \nu \). Each firm \( j \) chooses one price for each of its brands, i.e., a vector of prices \( p_j = (p_{j1}, \ldots, p_{j(#K_j)}) \), simultaneously with all other firms. We can show:

**Proposition 3** A pricing equilibrium exists for which \( p_j = r \) for all \( j \notin \{1, 1_2\} \), and cumulative distribution functions \( F_{j1} \) and \( F_{j2} \) for prices \( p_{j1} \) and \( p_{j2} \) for brands \( 1_1 \) and \( 1_2 \) as in (2) and (3), with \( n_1 \) and \( n_2 \) being replaced by \( n_1 (= n_i) \) and \( n_2 (\geq n_1) \), respectively.

**Proof.** We only sketch the proof. A full proof applies arguments which, in detail, are very similar to the arguments used in the proof of Proposition 2. Consider first firm 1. The optimization problem of firm 1, given the candidate equilibrium strategies of all other firms, is exactly equivalent to the problem of the single multi-brand firm in Proposition 1 to find the optimal reply, given that \( F_{j1}(p_{j1}) \) for brand \( 1_z \), and \( p_j = r \) for all other brands \( j \in (S \setminus (K_1 \cup \{1_2\})) \), and the optimal reply is exactly the one described in Proposition 2.

Turn now to the other multi-brand firms \( j \). Consider first a firm \( j > 2 \). Given the cumulative distributions

\[
F_{j1}(p) = 1 + \frac{n_1}{m} \left(1 - \frac{r}{p}\right) \text{ for } p_1 \in \left[\frac{n_1 r}{n_1 + m}, r\right],
\]

(9)

\[
F_{j2}(p) = 1 + \frac{n_2}{m} \left(\frac{(n_1 + m)n_1 r}{(n_1 + m)mp}\right) \text{ for } p_2 \in \left[\frac{n_2 r}{n_1 + m}, r\right],
\]

(10)

\[
F_{j1}(p) = F_{j2}(p) = 0 \text{ for } p \in [0, \frac{n_2 r}{m}) \text{ and } F_{j1}(p) = F_{j2}(p) = 1 \text{ for } p \geq r,
\]
and given $p_j = r$ for all brands $j \notin (\{1, 2\} \cup K_j)$, we confirm that any vector $p_j \neq (r, r, \ldots, r)$ yields a lower payoff than the price vector $(r, r, \ldots, r)$. For any $\hat{p}_j$ with $\hat{p}_{i_j} < r$ for $i_j \neq 1_j$, firm $j$ can increase its profit by choosing $p_j$ which is identical with $\hat{p}_j$ in all components except in component $i_j$, where $\hat{p}_{i_j} < r$ is replaced by $p_{i_j} = r$. To see this, note that a change to $p_{i_j} = r$ cannot lead to a lower sales revenue on any of $j$’s brands other than $i_j$, but the sales revenue on $i_j$ for $\hat{p}_{i_j} < r$ is at most equal to

$$
(1-F_{i_j}(\hat{p}_{i_j}))(1-F_{i_j}(\hat{p}_{i_j}))m_1 + \hat{p}_{i_j}n_{i_j} < r n_{i_j}
$$

by $n_{i_j} > n_{i_j}$, analogously to the reasoning in (6).

Finally, consider firm $j = 2$, given $F_i(p_{i_2})$ and $p_i = r$ for all $i \in (S \setminus K_2 \cup \{1\})$. Again, it can be shown that for any $\hat{p}_2$ with $\hat{p}_{i_2} < r$ for $i_2 \neq 1_2$, firm 2 can increase its profit either by a straightforward increase in $p_{i_2}$ to $p_{i_2} = r$ (which is the case if $\hat{p}_{i_2} \neq \min \{\hat{p}_{i_2}, \ldots, \hat{p}_{i(K_2)}\}$, or by simultaneously replacing $\hat{p}_{i_2}$ with $\hat{p}_{i_2}$ and by increasing $p_{i_2}$ to $p_{i_2} = r$. This way it can, again, be argued that any optimal reply needs to be of the format $(p_{i_2}, r, r, \ldots, r)$. From here, the optimizing problem of firm 2 is reduced to the optimal choice of $p_{i_2}$, and it is analogous to the proof in Proposition 2 to see that any $p_{i_2} \in \left[\frac{n_{i_2}}{n_{i_2} + m}, r\right]$ is optimal.

In other words, in the equilibrium with several multi-brand firms, the prices of all brands are equal to the consumers’ reservation prices, except for the prices of two brands. These two brands are owned by different firms, and one of the two brands is the weakest among all brands. By the notation used here, this weakest brand is $1_1$. The other brand is $1_2$; it is owned by firm 2, and it is the weakest brand among the brands owned by firm 2. Note that $1_2$ can be a much stronger brand than most of the brands owned by firm 1, and it need not be the second weakest brand among all brands. Actual competition for the non-loyal customers occurs through these two brands. The key to the proof of Proposition 3 is the observation that the optimal reply $p_{k_{i_j}}$ of the multi-brand firm in the equilibrium given the pricing behavior of all other firms depends only on the prices chosen by these firms, but not on whether the prices for all these brands are chosen by a large number of single-brand firms, or by a
smaller number of multi-brand firms.

Taking Proposition 3 as the point of departure, we can address the question of the profitability of a merger. For this purpose note that the equilibrium payoffs of all multi-brand firms \( j \geq 2 \) are equal to

\[
\sum_{i \in K_j} r_{n_i}.
\]

The payoff of the firm owning brand \( 1 \) equals

\[
\frac{n_{1i} + m}{n_{1i} + m} r_n + \sum_{i \in K_j \setminus \{1\}} r_{n_i}.
\]

Inspection of these expressions shows our key result: a merger is profitable only if the merging firms hold brands \( 1 \) and \( 2 \), or, in words:

**Proposition 4** A merger between multi-brand firms that does not lead to a monopoly increases the sum of the merging firms’ payoffs if and only if this merger includes firms owning the brands for which the equilibrium prices are lower on average than the reservation prize for loyal consumers in the pricing equilibrium without merger.

Summarizing, we found that merger is profitable for firms if these firms own the two brands for which a deviation from \( p_j = r \) is optimal in the equilibrium without merger, that is, if the firms who own the brands which actively compete for the non-loyal customers merge. While the merger will generally not eliminate competition for the non-loyal customers, it will relax this competition, because this competition will involve a stronger brand than in the absence of the merger, and this stronger brand has a higher opportunity cost of competing for the non-loyal customers.

**III. CONCLUSIONS**

Brand loyalty is an important element of firms’ price competition. We
consider how ownership of multiple brands affects the outcome of Bertrand competition with many loyal customer groups and with a group of price-sensitive non-loyal customers. Our main research question is how profits are affected by mergers and acquisitions, if the acquiring firm keeps the brands and acquires the group of loyal customers with this brand. We find that many types of merger and the brand portfolio reallocations they imply are neutral as regards their strategic aspects for market competition. However, we also identify mergers and acquisitions that reallocate brand portfolios in a way that has strategic effects for the market competition outcome. Particularly if firms with weak brands absorb other weak brands, this may shield these weak brands and relax competition among weak brands. It also draws stronger brands into the competition for non-loyal customers. Our results contribute a strategic market-interaction-based explanation to why some firms acquire large conglomerates of brands.

It is interesting to compare our analysis with the analysis by Baye, Crocker and Ju (1996), as they also refer to car producers with many brands in a merger context. They use GM as an example of multiple, mutually competing divisions under the umbrella of a holding company for the possibly beneficial strategic effects of the creation of multiple decision units inside a firm that compete both with other firms and among each other. Their claim is that GM and other firms used a strategy of divisionalization to generate an effect that just reverses the effect of the merger: holding companies consisting of multiple firms that compete with each other can attract a larger share of total industry profit than one single monolithic firm with a fully coordinated policy with quantity competition. This increase in market share may dominate the reduction in industry profit as a whole. Their theoretical result, considering divisionalization as the inverse of merger, is intellectually appealing. However, their divisionalization argument captures only one part of the story of the US car industry. Historically, the creation of multi-brand firms such as GM or Chrysler is not mainly the result of a firm splitting its operation into several divisions. GM was the result of a merger of several smaller car producers, and many of its brands, such as Pontiac, Cadillac, Hummer, or Opel were acquired rather than newly generated. An industry structure
dominated by the big three, GM, Chrysler and Ford, is mainly the outcome of a wave of new firm entries, followed by a process of acquisitions, mergers and exits. Where firms acquired a firm with another brand, they often kept and preserved the acquired brand.\textsuperscript{16}

We take account of the fact that many multi-brand companies are not the outcome of a process of divisionalization, but of a process of acquisitions, together with the policy to keep the acquired brands alive; hence, mergers and acquisitions need to be explained in many cases, rather than a split-up of firms in different, competing brands. In some cases the brand itself may have been the most valuable object acquired.\textsuperscript{17}

Our framework provides such an explanation.

\section*{IV. APPENDIX}

In the Appendix we prove part (ii) of Proposition 2.

\textbf{Proof.} For a proof of part (ii) it is sufficient to show that the strategies in Proposition 1 are mutually best replies if $\{1, 2\} \not\subseteq K$. Three cases need to be distinguished: $\{1, 2\} \cap K = \emptyset$ (case 1), $1 \not\in K$ but $2 \in K$ (case 2) and $1 \in K$ but $2 \not\in K$ (case 3). Note that, for all three cases, we can take $p_j < r$ for granted, as $p_j > r$ is clearly dominated by $p_j = r$ for all $j \in S$.

Consider first the case $\{1, 2\} \cap K = \emptyset$. Given that the single-brand firms’ strategies are optimal replies (which follows directly from Proposition 1), it is sufficient to show that, given $F_1$ and $F_2$ and $p_j$ for all $j \not\in K$ as in Proposition 1, the merged firm cannot do better than by choosing $p_{i_k} = r$ for all $i_k \in K$. If the multi-brand firm follows the strategy in the candidate equilibrium, the firm’s payoff is equal to

\textsuperscript{16}Klepper (2002), for instance, reports that the structure of the US car industry is an outcome of a consolidation process: while more than 500 firms entered into this market in its first 20 years, exits and acquisitions led to an industry which was dominated by GM, Ford and Chrysler, accounting for more than 80 percent of the output in the US car industry in the years after 1930. Klepper presents the acquisitions of Olds Motor Works, Cadillac and Chevrolet by GM as an illustrative example.

\textsuperscript{17}An example illustrating this claim is the struggle between BMW and Volkswagen over the takeover of Rolls-Royce/Bentley which was a firm with two strong brands; the struggle ended with each of them obtaining one of the two brands. Bahadir, Bharadwaj and Srivastava (2008) report that the value of brands is often a substantial fraction of the takeover price.
If the merged firm deviates and chooses any other joint distribution $F(p_K)$, the resulting payoff is lower. To confirm this consider any deviation $\hat{p}_K \neq (r,r,\ldots,r)$. Let $\hat{p}_{j_{min}}$ the smallest component in $\hat{p}_K$, with $\hat{p}_{j_{min}} < r$. Then the maximum payoff that may emerge from this choice for the merged firm is bounded from above by

$$\left(1-F_1(\hat{p}_{j_{min}})\right)\left(1-F_2(\hat{p}_{j_{min}})\right)\hat{p}_{j_{min}} m + \sum_{j_k \neq K} \hat{p}_{j_k} n_{j_k} \tag{A2}$$

$$\leq \left(1-F_1(\hat{p}_{j_{min}})\right)\hat{p}_{j_{min}} m + \hat{p}_{j_{min}} n_{j_{min}} + \sum_{j_k \neq K \setminus \{j_{min}\}} n_{j_k} r$$

$$< \sum_{j_k \in K} n_{j_k} r$$

The latter inequality follows from inserting $F_1(p)$ as in (2) and $n_{j_{min}} > n_2$.

Consider next the case with $2 \in K$, $1 \notin K$. Again, if the multi-brand firm chooses the same pricing strategy as a collection of single-brand firms owning these brands, then the remaining single-brand firms' strategies are individually optimal replies to this strategy by Proposition 1. Consider the multi-brand firm given $F_1(p)$ as in (2) and given $p_j = r$ for all $j \in S \setminus (K \cup \{1\})$. Let $i_K \in K$ be sorted by increasing brand strength. Then $i_K = 2$. We first show that, for any $\hat{p}_K$ with $p_{i_k} \neq r$ for $i_k \neq 1$, a price vector exists that yields higher profits. To see this, several cases need to be distinguished. If $\hat{p}_{i_k} = \min\{\hat{p}_{i_k}, \hat{p}_{i_k}, \ldots, \hat{p}_{i_K}\}$, then a simple increase of $p_{i_k}$ from $\hat{p}_{i_k} = \hat{p}_{i_k}$ to $p_{i_k} = r$ increases the multi-brand firm’s profit by $(r - \hat{p}_{i_k}) n_{i_k} > 0$. If $\hat{p}_{i_k} = \min\{\hat{p}_{i_k}, \hat{p}_{i_k}, \ldots, \hat{p}_{i_K}\}$ for $i_k \neq 1_K$, then the following changes in components of $\hat{p}_K$ increase the firm’s profit: an increase from $p_{i_k} = \hat{p}_{i_k}$ to $p_{i_k} = r$ combined with a decrease from $p_{i_k} = \hat{p}_{i_k}$ to $p_{i_k} = \hat{p}_{i_k}$ increases profits by at least $(r - \hat{p}_{i_k}) n_{i_k} - (\hat{p}_{i_k} - \hat{p}_{i_k}) n_{i_k} > 0$. This shows that any optimal price vector must be of the form $(p_{i_k}, r, \ldots, r)$. But for this set of price vectors, given $F_1(p)$, any $p_{i_k} \in \left[\frac{n_{r^2}}{n_2 + m}, r\right]$ yields the same payoff and this payoff is higher than for any $p_{i_k} > r$ or for $p_{i_k} < \frac{n_{r^2}}{n_2 + m}$.
Consider finally the case with \( 1 \in K, \ 2 \not\in K \). Again, if the multi-brand firm chooses the same pricing strategy as a collection of single-brand firms owning these brands, then the remaining single-brand firms’ strategies are optimal replies to this strategy by Proposition 1. Consider therefore the multi-brand firm for given pricing strategies \( F_2(p_2) \) as in (3) and \( p_j = r \) for all \( j \in S \setminus (K \cup \{2\}) \). Let \( i_K \in K \) be sorted by increasing brand strength, such that \( 1_K = 1 \), and \( 2_K > 2 \). We first show that, for any \( \hat{p}_k \) with \( p_{i_k} \neq r \) for \( i_k \neq 1 \), a price vector exists that yields higher profit. To see this, several cases need to be distinguished. If \( \hat{p}_k = \min \{ \hat{p}_{i_k}, \hat{p}_{2_k}, \ldots, \hat{p}_{s_k} \} \), then an increase of \( p_{i_k} \) from \( p_{i_k} = \hat{p}_{i_k} \) to \( p_{i_k} = r \) for \( i_k \neq 1 \) increases the multi-brand firm’s profit by \( (r - \hat{p}_{i_k}) n_{i_k} > 0 \). If \( \hat{p}_k = \min \{ \hat{p}_{1_k}, \hat{p}_{2_k}, \ldots, \hat{p}_{(\#K-1)k} \} \) for some \( i_k \neq 1 \), then an increase from \( p_{i_k} = \hat{p}_{i_k} \) to \( p_{i_k} = r \) combined with a decrease of \( p_{i_k} \) from \( p_{i_k} = \hat{p}_{i_k} \) to \( p_{i_k} = \hat{p}_{i_k} \) increases profits by at least \( (r - \hat{p}_{i_k}) n_{i_k} - (\hat{p}_{i_k} - \hat{p}_{i_k}) n_{i_k} > 0 \). This shows that any optimal reply must be of the form \((p_{i_k}, r, \ldots, r)\). But for this set of price vectors, given \( F_2(p_2) \), any \( p_{i_k} \in [\frac{n_j r}{n_j + m}, r) \) yields the same payoff and this payoff is higher than for any \( p_{i_k} \geq r \) or for \( p_{i_k} < \frac{n_j r}{n_j + m} \). \( \blacksquare \)
References


