The bargaining family revisited*

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Abstract

We suggest a family bargaining model where human capital investment decisions are made noncooperatively in a first stage, while day-to-day allocation of time is determined later through Nash bargaining, but with noncooperative behavior as the fall back. One finding is that overinvestment in education may be even more of a problem in such a semi-cooperative model than in a fully noncooperative one. Even though both the semi-cooperative model and the fully noncooperative model predict overinvestment in education, policy conclusions that follow from the two models are distinctly different.

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1 Introduction

This paper suggests a bargaining model of the family that mingles cooperative and non-cooperative elements. At the outset of marriage the spouses choose their investment in education. These investments have a long-lasting effect on spouses' labor market productivities during marriage and are assumed to be made noncooperatively. Later, during day-to-day family life, family members negotiate with each other about the allocation of their time between paid work in the outside labor market and household production of a public good. The fall-back position in this household bargain, though, is noncooperative behavior also in the second stage. To what extent is the allocation that follows from this model set-up inefficient? We find that the incentives to overinvest in education geared towards the outside labor market might be even stronger than in a fully noncooperative model.

Economic theorizing about family behavior standardly assumes that the family is able to reach efficient outcomes. For instance, in his seminal work in family economics Gary Becker (e.g. 1991) for the most part works with this assumption.

Reuben Gronau's well-known work on the allocation of time in the family (1973) is an efficiency model. In efficiency models of the family it is sometimes postulated that the family maximizes a joint 'welfare function'. This, however, should only be seen as a convenient tool for generating efficient outcomes – not as an attempt to describe actual family behavior. Rather, the heart of Becker's and Gronau's efficiency approach is that distributional concerns are neglected, and that any distributional fight between family members is seen not to influence the efficiency of outcomes.[1]

Lundberg and Pollak (1993, 1994) and Konrad and Lommerud (1995) use noncooperative game theory to study a family's allocation of time between market work and household production of a family public good.[2] Fully noncooperative behavior is hopefully rare in family contexts, apart from families at or beyond the brink of divorce. Noncooperative models should perhaps most fruitfully be thought of as an alternative benchmark in family modeling. The efficiency model gives us the outcome when complete family contracts can be enforced, the noncooperative model when no contract can be enforced at all. The truth probably lies in between.[3]

Both Lundberg-Pollak and Konrad-Lommerud, and many other authors that have used noncooperative game theory in the theory of the family, point at the possibility of using the noncooperative equilibrium of such models as the fall-back in household bargaining theory – and that this might be a main justification for the study of noncooperative family models. Binmore, Rubinstein and Wolinsky (1986) seek to give the axiomatic Nash solution an underpinning through noncooperative game theory. They conclude that fall-back utilities should be interpreted as 'utilities during conflict' rather than the utilities that can be obtained by leaving the relationship altogether. The utilities from noncooperative behavior might better capture this idea of 'utility during conflict' than the utilities as singles - the latter is a much used fall-back point in the family bargaining literature. Below we will emphasize that the empirical predictions from the two alternative fall-back points will be quite similar.

Lundberg and Pollak go some way towards implementing the idea of noncooperative behavior as the fall-back in bargaining, but they use a somewhat special model where it is assumed that some activities are only done by men, others only by women. Here we want to study a cooperative family model with a noncooperative fall-back, but without forcing the model only to pay attention to such corner solutions.[4]

Further, we consider educational investments made prior to the family bargain. We assume that these investments are made noncooperatively. This means that the model no longer is a full efficiency model – Nash bargaining now only achieves constrained efficiency taking educational investments as given. We argue that it is far less likely that decisions that have a long-lasting impact and are made at the beginning (or even before the start) of a relationship will be made as cooperative decisions. We therefore think that a model that mingles cooperative and noncooperative elements in the way suggested may be quite realistic. In family models that allow for inefficient family decision making – this applies both for 'transactions costs' type of models where one imposes exogenous limitations on what can and what cannot be contracted upon, and for fully noncooperative models – a typical feature is that investing in skills compatible with providing a family public good and foregoing to invest in building up one's capacity to earn market income, might be a very vulnerable strategy. This is also the case here. Both for the case where later interaction in the family is characterized by full noncooperation and when the family bargains with the noncooperative equilibrium as fall-back, there is overinvestment in education relative to a first-best, complete contracts setting. But as we shall see, the reason behind the overinvestment is quite different in the two cases. And, perhaps surprisingly, the overinvestment problem might very well be worse with (constrained) efficient bargaining.

The remainder of this paper is organized as follows. Section 2 is divided in two parts: In section 2.1 we find the noncooperative equilibrium and the Nash bargaining solution with this equilibrium as fall-back – for the case that the labor market productivities of the two family members are given. In section 2.2 we investigate how the fact that people anticipate to participate in this type of interaction later on influences their investments in education. Section 3 offers a brief summary of the paper.

2 The model

We consider a family, consisting of two persons, $i = f, m$. Each of them has a payoff function

$$u^i = x_i + G - a(g_i) - b(w_i). \tag{1}$$

Here, G is the amount of a family public good; it is the sum of individual time contributions of the two, that is,

$$G = g_f + g_m. \tag{2}$$

Examples of the family public good are well brought-up children, a clean and tidy home, a beautiful garden, or the well-being of elderly parents.[5] For simplicity we assume that both family members care equally much for the family public good.

Further, x_i is the individual's consumption of private goods and is determined as

$$x_i = w_i(y - g_i) \tag{3}$$

where w_i is i's labor market wage[6] – to be determined below, and y is i's maximum time available for working in the labor market and for contributions to the public good.[7] Apart from reducing the time available for labor market activities, contributing to the public good has some 'psychic' cost that is measured by a convex cost function $a(g_i)$. While the individual has low marginal psychic cost (perhaps even negative) of contributing the first few units to the family public good, individuals increasingly dislike to spend more time on these activities. Finally, the model endogenizes the labor market wage rate. Individuals can spend effort on educational activities that increase their labor market wage. This effort is denoted $b(w_i)$. We assume that the marginal cost of increasing the market wage is positive and increasing, that is, $b'(w_i) > 0$, and $b''(w_i) > 0$.[8] One would naturally think of educational effort as consisting both of monetary and psychic terms. Even though we have here stressed the psychic element, we assume for simplicity that effort is observable and measurable in monetary terms.

We have chosen to work with the special functional form (1) of the utility function. The main reason for this is simplicity. With linearity of income the Nash bargaining solution takes on a simple and appealing form: Each party to the bargain gets his or her fall-back utility level plus half of what surplus is created in the relationship above the sum of fall-back utilities.[9] Utility as in (1) also makes the strategic interaction between the parties in a private-contributions-to-a-public-good game quite simple. It turns out that when the spouses act noncooperatively, even though their utilities are intertwined through the existence of a family public good, each partner's behavior is independent of the other partner's behavior. This simplicity is not necessarily a disadvantage. It makes the noncooperative outcome a 'clearer' benchmark for understanding the family bargaining model: When there is strategic interaction only in the cooperative case, this shows that the bargaining process itself creates the incentives for strategic thinking and these incentives should carry over qualitatively if individuals' preferences are not linear.

We consider the following two-stage games. Individuals have payoff functions (1). In stage 1, individuals simultaneously choose their labor market wage (education decision). In stage 2, individuals know the choices of wage rates in stage 1 and simultaneously decide on how much time they devote to activities that contribute to the family public good. We assume throughout that the education decison is a noncooperative choice. Choice of education often has a once-and-for-all character. Hence, repeated interaction between family members cannot be used as an argument for efficient bargaining to arise. Moreover, many important educational decisions take place in an early stage of the relationship – perhaps even before the family is formed – and prior to the decades of marriage when the contributions to the family public good are to be made. It is plausible that attempts to cooperate on the education decision may fail.

The contribution decision in stage 2 may be the outcome of a noncooperative game; but since this contribution game typically is repeated it may be more reasonable to assume that the couple will find a cooperative outcome regarding the contribution decision in stage 2.

We now solve the problem backwards, starting with a situation with given educational choices (w_f, w_m). We first calculate the stage-two Nash contribution equilibrium in case no cooperation in stage 2 occurs. Then we use this outcome to calculate the Nash bargaining solution for which the noncooperative equilibrium is considered to be the threat point. Having solved these problems contingent on given educational choices, we determine the stage-1 subgame perfect equilibrium choices of labor market productivities for the two games we will denote N and C: Game

N (for 'no commitment') is the game with non-cooperation in stage 2, game C (for 'cooperation') is the game with Nash bargaining in stage 2.

2.1 Stage-two outcomes

A noncooperative Nash equilibrium in stage 2 for given educational choices (w_f, w_m) is a pair (g_f^*, g_m^*) such that g_f^* maximizes

$$u^f = w_f(y - g_f) + g_f + g_m^* - a(g_f) - b(w_f) \tag{4}$$

for $g_f \in [0, y]$, and analogously for m and $g_m \in [0, y]$. The payoff functions u^i are continuous in (g_f, g_m) and concave in g_i; strategy spaces $g_i \in [0, y]$ are compact, convex, non-empty sets. Therefore, a Nash equilibrium in pure strategies exists. Contribtions that maximize i's payoff do not depend on the contributions by the other family member, and strict convexity of $a(g_i)$ makes these contribution choices unique for given w_i. Hence, the equilibrium is unique. Assuming an interior solution, the equilibrium contributions are determined by the first order conditions

$$-w_i + 1 - a'(g_i) = 0. \tag{5}$$

Let g_i^* be the solution of (5). We have

$$g_i^{*\prime} \equiv \frac{dg_i^*}{dw_i} = -\frac{1}{a''} < 0 \tag{6}$$

by the convexity of $a(g_i)$, and $dg_i^*/dw_j = 0$ for $i \neq j$.

The equilibrium utility levels in the noncooperative stage-two game will be denoted $u^{i*} \equiv u^i(g_f^*, g_m^*)$.

We now turn to characterizing the efficient outcome. We assume that monetary transfers between family members are allowed. The efficient levels of contributions to the public good, denoted by g_f^e and g_m^e , are implicitly determined by

$$w_f + a'(g_{_f}) = w_m + a'(g_{_m}) = 1 + 1, \tag{7}$$

assuming that the time budget constraint $g_i^e \leq y$ is not binding.

The first equation describes the fact that marginal cost of contributions to the public good have to equate, and the second equation is the Samuelson condition for public goods provision by which marginal cost of production have to be equal to the sum of willingnesses to pay.

The amounts g_f^e and g_m^e are uniquely determined and are only a function of the individuals' own wage rate w_f and w_m, respectively, with $g_i^{e\prime} \equiv dg_i^e/dw_i = (-1/a'') < 0$ and $dg_i^e/dw_j = 0$ for $i \neq j$. Further, from (5), (7) and $a'' > 0$, we get

$$g_i^e(w_i) > g_i^*(w_i) \tag{8}$$

This result is stated as a proposition:

Proposition 1 *In the noncooperative Nash equilibrium, a joint increase in contributions to the public good would be a Pareto improvement.*

The simple game described so far in this section is an example of a voluntary-contributions-to-a-public-good game.[10] Underprovision of the public good is typical for this class of models. The model as it is specified implies that one person's contribution to the family public good is determined solely by his or her own productivity in the labor market – the productivity of the other member of the family does not matter. Our framework is not targeted to study specialization according to comparative advantages, an issue that is central in much previous work on the time allocation of family members.[11]

Suppose now that the individuals solve the contribution problem via Nash bargaining using the noncooperative equilibrium utility levels as the threat point of the bargaining game. We assume that monetary side-payments can be made between the family members. Within this framework efficiency will be realized, so what remains to be studied is intrafamily distribution. Since utility is linear in income and since efficient contributions g_f^e and g_m^e only depend on w_f and w_m as in (7) but not on the final utility outcomes, the utility possibility frontier is $u^f = V - u^m$, with

$$V \equiv u^f(g_f^e, g_m^e) + u^m(g_f^e, g_m^e) = \sum_{i \in \{f,m\}} [(y - g_i^e)w_i - a(g_i) - b(w_i)] + 2(g_m^e + g_f^e) \quad (9)$$

The utility possibility frontier is linear and has slope minus one. The Nash bargaining solution yields utilities

$$u_{NB}^i = \frac{V}{2} + \frac{u^{i*} - u^{j*}}{2} \text{ for } i, j \in \{f, m\}, \text{ and } i \neq j. \quad (10)$$

Equation (10) shows that anything that influences utility differences in the noncooperative equilibrium will influence intrafamily distribution.[12] The model predicts that to have better outside opportunities tilts intrafamily distribution one's way – a prediction that also can be generated within a Nash bargaining model with utilities as singles as fall-back. The reason why the models become so similar is that noncooperation has many common traits with divorce. One can think of noncooperation as 'internal divorce', that is, divorce without leaving the household. Noncooperation means that all decisions are taken to maximize one's own utility – transfers of money cease and contributions to the public good are set at a level that does not take into account the benfits to the other partner. The difference between noncooperation and divorce is that the parties in the former case still live together so the family public goods are still public goods. After a divorce some goods, as a tidy home, cease to be public goods at all – whereas others, as children, remain public goods, but the valuation of the 'goods' may have changed for the leaving party.[13] We are lead to conclude that -operating at a high level of abstraction- the empirical predictions of using noncooperation as the fall-back are surprisingly similar to the case with utilities as single as fall-back.[14]

2.2 The educational choice

Consider now the equilibrium choice of education in stage 1. The equilibrium choice will in general depend on whether stage 2 is characterized by noncooperation or by cooperation. We consider the two cases one by one.

First we consider the problem for the case without cooperation in stage 2, that is, for game N. As mentioned, there is little strategic interaction in stage 2. Individual i's choice of own wage does affect i's equilibrium contributions to the public good, and, hence, the other individual's utility. However, the contribution choice of the other individual does not depend on i's anticipated contribution choice; this makes the other individual's education choice independent of i's education choice. More formally, f's problem is to choose a wage that maximizes

$$u^f = w_f[y - g_f^*(w_f)] + g_f^*(w_f) + g_m^*(w_m) - a(g_f^*(w_f)) - b(w_f) \tag{11}$$

where $g_i^*(w_i)$ is determined by (5). The choice of w_f that maximizes (11) is independent of w_m and is determined by the first-order condition $\partial u^f/\partial w_f = 0$ which, using (5), can be written

$$y - g_f^*(w_f) = b'(w_f) \tag{12}$$

and analogously for m. For an interior maximum, the marginal cost of the last unit of wage increase equals the additional private income that is generated by this wage increase.

Conditions (5) and (12) are necessary conditions for an interior utility maximum and determine a subgame perfect equilibrium with equilibrium values denoted $w_f^*, g_f^*, w_m^*, g_m^*$. We will assume that the conditions are also sufficient and determine a unique maximum.[15] The following proposition characterizes the efficiency properties of such an interior subgame perfect equilibrium. A proof is in the appendix.

Proposition 2 *In the N-game, a joint decrease in education investment that leads to a reduction in both individuals' wages by one marginal unit would be a Pareto improvement.*

Consider a small proportional tax t_w on educational expenditure. This tax changes the cost function $b(w_i)$ to $(1 + t_w)b(w_i)$. By convexity of b and by (12), a small tax leads to a small decrease in w_i^* for $i = f, m$. This decrease is Pareto improving by Proposition 2. Accordingly, we find

Corollary 1 *In the N-game, a small proportional tax on education expenditure with lump-sum redistributed revenues is Pareto improving.*

The noncooperative subgame perfect equilibrium (equilibrium of game N) suffers from underprovision of the family public good, and a joint decrease in education effort is welfare enhancing. The driving force behind this result is basically the same as when noncooperation in stage 2 was analyzed for given labor market productivities. In stage 2 there is underprovision because the family members do not take into consideration the other's gain from one's own contribution to the public good. This is the case also in the full two-stage noncooperative game N: But here we in addition get that people – knowing they will work much in the labor market in stage 2 due to noncooperation – adapt to this by investing more in increasing their labor market productivity. The overinvestment in education in this game only follows from the inefficiently high labor supply in stage 2; investments in education are efficient relative to the expected low level of the production of the family public good. In particular, in game N the individuals have no strategic motive for investing in education – meaning that here it is not possible to influence the behavior of the other by altering one's own educational level.

Taxing education to bring about a Pareto improvement is – in game N – a typical second-best policy: One introduces a second distortion (the human capital distortion) to correct a primary one (the underprovision due to the mutual external effect of contributions to the family public good). We postpone a further discussion of policy options till after the analysis of game C, the game with Nash bargaining in stage 2.

For game C, the following proposition holds.

Proposition 3 *Let the efficient education decisions and contributions with full cooperation in both stages be determined by $\partial V/\partial w_f = \partial V/\partial w_m = 0$ with V defined in (9). In game C both individuals have an incentive to choose an education higher than the efficient education.*

A proof is in the appendix. The intuition for the incentive for overinvestment can be seen from (10). Nash bargaining with linear payoff functions implies that the individuals get what they have in the threat point, plus half of the possible gains from cooperation. Imagine first that the gains of cooperation are not changed by an action chosen by one of the players. But this action changes utilities in the threat point. Suppose that both utilities in the threat point decrease, but the utility of the individual who chooses the action decreases by less than the utility change in the threat point for the other individual. Although overall resources that can be distributed between the two would not change in this case, the action improves the final situation for the individual who carried out this action. But moreover, an action can even be desirable from the perspective of the individual although it reduces the sum of payoffs, if it increases the difference between the two individuals' utilities in the threat point sufficiently more.

An individual chooses w_i that maximizes (10). In order for $\partial u^i_{NB}/\partial w_i = 0$, investments in education in fact will be driven to the point where the sum of utilities is decreased. Individuals know that if they overinvest, they waste resources from a social point of view and reduce the overall size of the cake that can be distributed. However, they increase their individual payoff in the noncooperative contribution game (their threat point) relative to the outcome for the other player.

The overinvestment results for games N and C do not necessarily generalize to more general specifications of utility. For game N, an increase in an individual's wage rate may increase the individual's contribution to the public good due to an income effect, leading to some strategic considerations. In game C, income effects would also add a strategic incentive to invest less. For some utility functions even underinvestment in education may result in both of these games. We abstracted from income effects to isolate some robust incentives for overinvestment. However, the overall results on investment become ambiguous with more general utility functions.

In the present context we have assumed that the marriage matching process is exogenous. For instance, people marry for passion or love, in which case the level of education does not influence the mating process. We think this assumption has some merit. However, abstracting from love, noncooperative family models often suggest that "likes" marry "likes", that is, mating is assortative. This would imply that having much education increases the probability that one will be married to someone also with much education. In turn, this would reduce the incentive to stategic overinvestment in education.[16]

In the economic theory of labor-management bargaining many models with a similar two stage structure as ours predict underinvestment in the first stage. A well-known reference is Grout (1984). A key assumption in this kind of model is that utility is additively separable over time. This means that if you undertake a large investment at a given period, this does not lower your utility the next period – so in a bargaining following somewhat later than the investment period the investment cost will be seen as irrelevant. We use a life-time utility concept, where the income effects of previous investment are not suppressed as in Grout. In our opinion the assumption of additively separable utility over time is more relevant in the theory of the union than in the theory of the family: The owners and workers of a firm at one moment may very well be different individuals than those present at earlier periods; less likely so in a family.

Games N and C entail overinvestment in education, but the reasons for the overinvestment results are distinctly different. In game N the overinvestment in education was just a reflection of the fact that noncooperative behavior in the family later on will lead the family public good to be underprovided. Consider the decision of one individual, for instance the male. Knowing that he will work much in the outside labor market, a rational choice is to increase investments in education. In game C, bargaining in stage 2 is efficient, so naturally the overinvestment cannot be a result of inefficient family decision making in this stage. Rather, the overinvestment is strategic in the sense that – through improving his fall-back utility level – it makes her willing to agree to stage 2 money transfers that are more favorable to him. The situation is one with an arms race externality: When both persons overinvest in this way, the difference between fall-back utilities remains constant (at zero) – so in the end overinvestment only reduces welfare without altering the intrafamily distribution. We find that the equilibrium wages of both individuals can even be larger in the symmetric equilibrium in game C than if individuals do not cooperate in stage 2:

Proposition 4 *The equilibrium wages in game C can be larger, equal or smaller than in game N. If the marginal cost $a'(g_i)$ is convex (concave) then the equilibrium wages w_i^c in game C are higher (lower) than the equilibrium wages w_i^* in game N.*

The proof of the proposition is again in the appendix. Proposition 4 shows that even more overinvestment can occur if spouses know that they will cooperate in the future. A benchmark case is the one with quadratic cost functions, $a(g_i) \equiv (\alpha/2)(g_i)^2$ and $b(g_i) = (\beta/2)(w_i)^2$. For this case the equilibrium values are

$$w_i^c = w_i^* = \frac{\alpha y - 1}{\beta\alpha - 1} \tag{13}$$

Intuitively, a marginal increase in w_i starting from the equilibrium level w_i^* in game N has two effects in game C. As shown in the appendix in equation (A6), the total effect consists of three terms:

$$\frac{\partial V}{\partial w_i} = [y - g_i^e(w_i^*)] - b'(w_i^*) - g_i^{*\prime}(w_i^*). \tag{14}$$

The first two terms measure the efficiency effect. The increase in w_i increases labor market income for the household on each unit of time that i uses in the labor market in the cooperative game, and the increase in w_i also has some direct marginal education cost. The last term relates to redistribution according to the Nash bargaining game. If the male considers a marginal increase in his wage at w_i^*, by the definition of the equilibrium of game N, this does not change his utility u^{m*}. However, due to the increased wage rate, he will marginally change his contribution to household production by $g_i^{*\prime}(w_i^*)$, which in turn reduces the utility of the female by this amount. This is strategically advantageous for the male.

Since the factors determining the amount of overinvestment in the two games are different, there can be more or less overinvestment in game C compared to game N. If overinvestment is higher in the cooperative case, the total efficiency problem may very well be worse in the model with Nash family bargaining than in the fully noncooperative model.

Even though game N and game C share the prediction of overinvestment in education, the fact that the underlying logic behind the result differs also means that the two models suggest different policy conclusions.

First, assume that a policy maker can choose between taxing education or in some way stimulating stage 2 provisions of the family public good. The latter can be brought about for instance by reducing the opportunity cost of working at home by income taxation. In game N the two instruments are equivalent. This is not so in game C. Here the overinvestment in education in a sense can be thought of as the 'primary' distortion. Changing the opportunity cost of providing the family public good can never lead to a first-best situation. For instance, income taxation that is sufficiently high to reduce educational investment to its first-best level leads to oversupply of the public good compared to the first-best since the opportunity cost of contributions to the public good is distorted downwards. Through taxation of education, though, the first best can be achieved.

A problem with recommending increasing educational investment costs based on models as game N or game C is that this would decrease education both for individuals who later lived in a family characterized by full non-cooperation or by our Nash bargaining model – and for those who do not. Our guess is that full non-cooperation is a rarer real phenomenon than the bargaining situation described in game C. This in turn means that this word of caution against using these models as arguments for trying to reduce investments in education applies even more for game N than for game C. Moreover, for both models there might be other distortions in the economy that point towards too little education, so that the possible 'overinvestment' in market abilities pointed at here simply is welcome.

Sometimes it is claimed that the increased probability of divorce makes investments in skills compatible with household production vulnerable, pushing people towards investment in skills that can be used in the outside labor market instead. If possible, policy makers should therefore try to increase marital stability. If we for the moment interpret the fully noncooperative setting in game N as the situation after a divorce, with the bargaining model of game C as the situation without divorce, this line of reasoning need not be correct. As we just have noted, overinvestment in building up abilities compatible with market work can in fact be larger in game C than in game N, so if a 'strengthening of marital ties' means to increase the probability that we are in a game C situation rather than in a game N situation, the result may well be more overinvestment.

It should also be noted that the model is one of equality between the sexes: There is no intrinsic reason why one sex rather than the other should invest more in education. 'Reducing education incentives' refers to doing so for both sexes.

3 Conclusions

The ruling tradition in family economics has been to assume that the family always is able to reach efficient outcomes. Both Becker type models of harmony as well as Nash bargaining models belong to this category. As a contrast to the efficiency models, it has lately been suggested to use noncooperative game theory in the study of family members' behavior for at least to admit the possibility that distributional conflicts can hurt efficiency. We have here suggested a family bargaining model that mingles cooperative and non-cooperative elements. Long-term decisions at the outset of marriage (or even only in anticipation of marriage) are taken noncooperatively. The later day-to-day management of the family is reached in (constrained) efficient bargaining, but with noncooperative behavior as the fall-back. A main finding is that within this semi-cooperative model with noncooperation as fall-back the incentives for inefficiently large investments in education might, in fact, be even worse than in a fully noncooperative model.

Moreover, as the reasons for overinvestment in market oriented human capital in the semi-cooperative model and the fully noncooperative model are distinctly different, this suggest different policy conclusions in the two models. In the fully noncooperative model, the basic problem is underprovision of family public goods, and, intuitively, a targeted policy of attacking this problem directly seems preferable to the circumvent alternative of discouraging education. Within the boundaries of the model, though, encouraging contributions to the family public good or discouraging market oriented education are equivalent policies. Not so in our semi-cooperative model. Here the contributions to the family public good are at the efficient level given investments in education. The overinvestment in education is the model's basic distortion. Encouraging provison of family public goods can by itself never lead to a first-best situation, discouraging education can. We also have noted in the passing that if the fully noncooperative model is interpreted as the situation after a divorce, while our semi-cooperative model is seen as a description of marriage, our results imply that one cannot be sure that a policy to 'strengthen marital ties' actually will reduce the overinvestment in market compatible skills.

4 Appendix

In this appendix we prove propositions 2 to 4.

Proof of Proposition 2. Suppose both individuals choose to decrease education cost according to the assumption in Proposition 2 such that $dw_f = dw_m = dw < 0$. At the initial equilibrium, with $w_f = w_f^*$ and $w_m = w_m^*$, we have

$$\frac{du^f}{dw} = [y - g_f^* - w_f^* g_f^{*\prime} + g_f^{*\prime} + g_m^{*\prime} - a'(g_f^*)g_f^{*\prime} - b'(w_f)] \underset{(5),(12)}{=} g_m^{*\prime} < 0 \qquad \text{(A1)}$$

and, analogously, $du^m/dw_{|w_m=w_m^*} = g_f^{*\prime} < 0$.

Proof of Proposition 3. Let $(w^e \equiv w_f^e = w_m^e)$ and $g^e(w^e) \equiv g_m^e(w_m^e) = g_f^e(w_f^e)$ be the efficient wages and contributions that maximize (9). The first-order condition determining the efficient wages,

$$\frac{\partial V}{\partial w_f} = \frac{\partial V}{\partial w_m} = 0, \qquad \text{(A2)}$$

can be rewritten as

$$y - g_i^e(w_i) - b'(w_i) = 0 \qquad \text{(A3)}$$

at $w_i = w_i^e$ for $i = f, m$.

If f chooses a wage that is marginally higher than the efficient wage, the change in her payoff is

$$\frac{1}{2}(\frac{\partial u_f^*}{\partial w_f} - \frac{\partial u_m^*}{\partial w_f}) = \frac{1}{2}(y - g_f^*(w_f^e) - b'(w_f^e) - g_f^{*\prime}(w_f^*)). \qquad \text{(A4)}$$

Now, $(-g_f^{*\prime}(w_f^*)) > 0$, and $y - g_f^*(w_f^e) - b'(w_f^e) \underset{(8)}{>} y - g_f^e(w_f^e) - b'(w_f^e) \underset{(A3)}{=} 0$. Hence, f's payoff is increasing in w_f at the efficient wage levels.

Proof of Proposition 4. As the problem is fully symmetric, without loss of generality we can concentrate on m's decision. The reasoning holds equivalently for f. The first-order condition that determines w_m^* is (12). The conditions that determine w_m^c is

$$\frac{\partial(2u_{NB}^m)}{\partial w_m} = 0. \tag{A5}$$

Whether $w_m^c \lesseqgtr w_m^*$ depends on whether $\partial(2u_{NB}^m)/\partial w_m \lesseqgtr 0$ at the equilibrium wage levels in game N. By (12), we get

$$\frac{\partial(2u_{NB}^m)}{\partial w_m}_{|w_m^*} = [y - g_m^e(w_m^*)] - b'(w_m^*) - g_m^{*\prime}(w_m^*), \tag{A6}$$

or, making use again of (12) to substitute for $b'(w_m^*)$,

$$\frac{\partial(2u_{NB}^m)}{\partial w_m}_{|w_m^*} = g_m^*(w_m^*) - g_m^e(w_m^*) - g_m^{*\prime}(w_m^*). \tag{A7}$$

[Figure 1 about here]

This sum in (A7) cannot be signed without further assumptions about the function $a(g_m)$, but it can be shown that the sign is positive (negative) if $a'(g_m)$ is convex (concave). This is illustrated for a convex $a'(g_m)$ in Figure 1. The figure shows how $g_m^e(w_m^*)$ and $g_m^*(w_m^*)$ are determined by (5) and (7). The distance between the horizontal lines $(2 - w_m^*)$ and $(1 - w_m^*)$ equals 1. Consider now a line tangent to $a'(g_m)$ at point A with slope $a''(g_m^*)$ and the intersection of this line with the horizontal line $(2 - w_m^*)$ in point B. The distance AC equals $1/a''(g_m^*)$ which, by (6), equals $(-g_m^{*\prime}(w_m^*))$. Since $a'(g_m)$ is convex, it intersects the horizontal line $(2 - w_m^*)$ to the left of B. Therefore, the distance AC is larger than the difference between $g_m^e(w_m^*)$ and $g_m^*(w_m^*)$.

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Footnotes

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[1] Nash bargaining models of the family (Manser and Brown, 1980, McElroy and Horney, 1981, Chiappori, 1988 and McElroy, 1990) and the 'collective' approach (Chiappori, 1992 and Browning and Chiappori, 1998) are examples of models that combine explicit recognition of the fact that the family consists of more than one decision maker with the assumption that outcomes are efficient.

[2] Other examples of models that use noncooperative game theory to picture family life include Leuthold (1968), Weiss and Willis (1985, 1993), Ulph (1988), Woolley (1988), Kooreman and Kapteyn (1990) and Browning (1997). For a careful discussion and overview see Bergstrom (1996).

[3] Some family models are of the 'incomplete contracts' vein, in that they (arbitrarily) assume that some contracts can be enforced, some others cannot. Examples of this 'transactions costs' literature include King (1982), Pollak (1985), Cohen (1987), Lommerud (1989) and Allen (1990).

[4] Even outside family economics, bargaining models where a noncooperative equilibrium represents the fall-back, are rare. One notable exception is Hoel's (1991) model of international environmental agreements. A reason might be that the formal analysis of such models can be quite difficult. Hoel's model is quite specialized, and also Lundberg and Pollak (1993) limit attention to the case of a Stone-Geary utility function. As will be seen, also here a restrictive assumption about the shape of the utility function will be employed.

[5] The latter example points to the fact that a 'family' need not be a married couple, but could for instance also be a brother and a sister.

[6] Note that we have set the productivity of using time to produce the public good at one, so w_i can be thought of as productivity in the labor market relative to that in household production.

[7] It is of course an exaggeration to think of the benefits from market activities as a pure private good, while the benefits from household activities constitute a pure public good. The flavor of our analysis would remain as long as the home produced good is more of a public good than the benefits from market work. Leuthold (1968) and Kooreman and Kapteyn (1990) work with the totally opposite assumption: They assume that household production (now refered to as 'leisure') is a private good while money earned in the market is a public good.

[8] We do not assume that increased education effort consumes time from the time budget y. Some educational effort in reality is of course time consuming. In our model structure, education effort takes place prior to the contribution game. Hence, this time effort would be spent in a time period different from the one in which the person has y available for working and for contributing to the family public good.

[9] The 'generalized Nash solution' allows for distributional shares to be α and $(1-\alpha)$, respectively, rather than fixing α at 0.5. It is straightforward to extend our model in this direction.

[10] Seminal references in the literature on this type of games are Warr (1982) and Bergstrom, Blume and Varian (1986).

[11] If major benefits of specialization exist, spouses could specialize according to comparative advantages. In this case, education investment can be interpreted as effort in a contest for becoming

the spouse who specializes in market activities.

[12] Note, though, that in this model where the two partners are identical, utilities under non-cooperation will be the same, so even under Nash bargaining there is even distribution of resources.

[13] It sounds quite cruel that a parent leaving the family should care less for the children, but the figures reveal, for example, that in Norway for children whose parents do not live together 30% wind up losing all contact with the father.

[14] Examples of empirical investigations about the effects of outside opportunities on a family's decisions are Grossbard-Schechtman and Neuman (1988), Rosenzweig and Schulz (1982) and Senauer and Jacinto (1988).

[15] This can be obtained for appropriate convexity and boundary conditions for the cost functions $a(g_i)$ and $b(w_i)$ and their first derivatives, and concavity properties of the utility function u^i.

[16] A full treatment of the marriage market should incorporate the possibility of comparative advantage, and possibly some elements of asymmetric information. This is left for future research.