Delegation in first-price all-pay auctions*

Kai A. Konrad[†], Wolfgang Peters[‡] and Karl Wärneryd[§]

August 6, 2003

[Short title: Delegation in Auctions]

Abstract

In a first-price all-pay auction buyers have an incentive to delegate the bidding to agents and to provide these agents with incentives to make bids that differ from the bids the buyers would like to make. Both buyers are better off in this strictly non-cooperative delegation equilibrium and the delegation contracts are asymmetric, even if the buyers and the auction are perfectly symmetric.

Keywords: first-price all-pay auctions, strategic delegation.

JEL classification numbers: D44.

---

*We thank an anonymous referee for valuable comments. The usual caveat applies. Konrad gratefully acknowledges a grant from DAAD.

[†]WZB and Department of Economics, Free University of Berlin. Please send correspondence to Kai A. Konrad, WZB, Reichpietschufer 50, D-10785 Berlin, Germany, fax: +49-30-25491-400, e-mail: kkonrad@wz-berlin.de.

[‡]Department of Economics, European University Viadrina.

[§]Department of Economics, Stockholm School of Economics.

1 Introduction

Suppose a buyer cannot go to an auction and make a bid. If she can send an agent, would she like to instruct the agent to make the same bid she would have made if she had been able to attend the auction? More generally, if all buyers can delegate their bidding decisions to agents, would they like to delegate the decision, and what incentives would they give to their agents in a fully non-cooperative equilibrium? The seminal discussion of Schelling (1960) on precommitment through delegation suggests that in many strategic situations, it may be beneficial to delegate to an agent with a different objective function than one's own.

In standard auctions there appears to be nothing to gain from this. However, this is not true in first-price all-pay auctions, pointing at an interesting difference between them. To confirm this we analyze such an auction in case of two opposing bidders: they make simultaneous bids for some indivisible good and the bidder with the highest bid is awarded the good, but all bidders have to pay their bid. Buyers behave non-cooperatively with respect to each other and delegate bidding decisions to agents. They give their agents incentives to make bids that differ from what buyers would like to bid if the buyers went to the auction. Even if the buyers are perfectly symmetric regarding their valuations of the auctioned good, the equilibrium contracts with their delegates are asymmetric: one buyer gives her agent a very strong incentive to make high bids, the other buyer gives her agent a more moderate incentive. It turns out that both buyers' payoffs are higher in these equilibria than without delegation.

The result is related to results in Wärneryd (2000), Baik and Kim (1997) and Kräkel and Sliwka (2002) who consider delegation in Tullock (1980) contests. In contrast to our results for first-price all-pay auctions, symmetric contestants will choose no delegates or symmetric delegation strategies in a contest. But similar to our result, delegation has a tendency for reducing the equilibrium effort.

All-pay auctions play an important role in many allocation processes. Some examples are campaigning (Skaperdas and Grofman 1995), contests in hierarchies and in the labor market (Rosen 1986, Glazer and Hassin 1988), military conflict (Hirshleifer 1995), lobbying (Ellingsen 1991, Nitzan 1994), redistributional contests in fiscal federalism (Wärneryd 1998) and contests in industrial organization when firms spend effort to win customers (Konrad 2000), or to become the de facto standard in product markets with network externalities (Besen and Farrell 1994). See also Dixit (1987) and Baik and Shogren (1992) for a more general discussion and Baye, Kovenock and deVries (1998) for a generalization of the class of contests, and for further applications.

The results in this paper apply to such contests, suggesting that we should observe delegation in these processes if delegation is feasible. For instance, firms contesting for favorable treatment by politicians may delegate the actual lobbying to professional lobbyists and provide the lobbyists with incentives quite different from their own incentives. Delegation may also contribute to explaining one of the puzzling results in the literature on lobbying.

It is a stylized fact that the sum of efforts that are observed in lobbying contests typically falls short of what the all-pay auction model predicts.[1] The result may also apply in markets in which sales and market shares are the result of contests, for instance, such as persuasive advertizing, or when firms contest for indivisible large scale projects. Two parent companies A and B who have affiliates a and b that compete on several markets may then think about the incentives they give to their affiliates. If A observes or anticipates that B will make a particular market the top priority for its affiliate b, if competition in this market has the nature of a contest, A may give its affiliate rather moderate incentives in this contest, instead of trying to even top the incentives given to b.

At this point we should emphasize that we do not consider collusive behavior among bidders. It is obvious that bidders can increase their payoffs if they succeed in collusive agreements that restrict their bids. In contrast, buyers in our analysis behave fully non-cooperatively and design their contracts with possible agents in order to maximize their own individual payoffs.

The structure of the paper is as follows. In section 2 we specify the timing of the delegation game, the type of contracts that can be used to delegate the choice of bids to agents, and the nature of the first-price all-pay auction. In section 3 we solve this game and characterize the subgame perfect equilibria. In section 4 we modify the assumptions about the types of feasible delegation

[1] For a discussion of the empirical evidence for this paradox in the contest of rent-seeking for an alternative explanation on the basis of free-rider incentives see, for instance, Katz, Nitzan and Rosenberg (1990) or Ursprung (1990).

contracts between buyers and their agents. In particular, we rule out that the contract may specify an upfront fee that is paid by the agent. It turns out that this restriction on feasible contracts may further increase the buyers' payoffs in the subgame perfect equilibria, and may also change the nature of the equilibria. Section 5 offers some conclusions.

2 The time structure of decisions

We consider the following three-stage game. There are four players: two buyers b_1 and b_2, and two agents, a_1 and a_2. Each buyer would like to obtain an indivisible good which is auctioned in a first-price all-pay auction. For simplicity both buyers have the same willingness to pay to obtain the good, denoted $B > 0$.[2] Agents attribute zero value to the good. These willingnesses to pay are commonly observed.

In stage 1, each buyer b_i, $i = 1, 2$, decides whether she hires agent a_i to represent her at the auction or whether she goes to the auction herself. If b_i hires agent a_i, she offers him a contract (φ_i, V_i). This contract specifies that agent a_i pays an up-front fee φ_i to his principal b_i when the contract is signed. If the agent obtains the good he has to sell this good to his principal at a prespecified price $V_i \in (0, \bar{V}]$. V_i is called the delegated valuation, because

[2] Our results can easily be generalized for cases where buyers' willingness to pay are not identical. We concentrate on the symmetric case for two reasons. The first reason is simplicity. Second, our main result is that delegation leads to asymmetric behavior of buyers that benefits both buyers. Asymmetry is a more interesting outcome if one starts with perfect symmetry than if the buyers are different from the very beginning.

this is the value the agent assigns to obtaining the good. Here, $\bar{V}$ is some non-negative and exogenously given finite upper limit.[3] The agent goes to the auction and pays his bids using his own money. If he does not win the good in the auction, neither him nor the buyer have any further obligations from the contract. The agent accepts the contract if and only if his payoff from accepting the contract is at least as high as his reservation utility of not participating in the game. His outside option is normalized to zero. If a delegation contract is signed between a buyer and her agent in stage 1, the terms of the contract are commonly observed by all players.

In stage 2 the bidders meet in a symmetric first-price all-pay auction. If a buyer has hired an agent, only her agent can go to the auction. If a buyer has not hired an agent, she goes to the auction herself. We will refer to party $i = 1, 2$ if we do not specify whether the buyer or her agent go to the auction. Each party makes simultaneous bids $e_i \geq 0$. These bids are

[3] The assumption that there is an upper limit for delegated valuations is essential for our results. Such an assumption is typically not made in the literature on contests or auctions, primarily because it is not needed: an interior equilibrium is obtained and the efforts or bids in equilibrium are not infinite. In our analysis, the choice of infinity would be desirable in the equilibrium. We think, however, that there is no such thing as a credible promise to pay an infinite amount: if the delegated valuation is infinitely high a buyer has a strong incentive to break the contract when the agent has won the auction. Even a severe punishment cannot prevent her from such behavior. Therefore, it seems to be a natural assumption that there is some upper limit for what principals can promise to pay. And we will not impose any restriction on the size of this upper limit.

monetary amounts which the bidder has to pay, regardless of whether the bidder wins the good or not. The good is allocated to the bidder who makes the higher bid. If both make the same bid, a coin is tossed to decide who gets the good. Hence, if $e_1 > 0$,[4] the probability that bidder 1 obtains the good is

$$p_1(e_1, e_2) = \begin{cases} 1 & \text{if} \quad e_1 > e_2 \\ 1/2 & \text{if} \quad e_1 = e_2 \\ 0 & \text{if} \quad e_1 < e_2. \end{cases} \tag{1}$$

Accordingly, the probability that the bidder of party 2 gets the good is $p_2 = 1 - p_1$. This completes stage 2 of the game.

The game is ended after stage 2 if party i who wins the auction is a buyer. If an agent wins the good, stage 3 describes the fulfillment of the contract between this agent and his principal. The agent who has obtained the good in the auction sells it to his principal for the agreed price V_i. No decisions have to be made in stage 3. We assume that the contract is enforced. Buyer i has to pay the agreed price V_i even if this exceeds her true valuation of the good.

Accordingly, if a buyer i does not delegate, her payoff is $p_i(e_1, e_2)B - e_i$. If a buyer delegates bidding to her agent, the buyer's payoff is

$$\pi_i = \varphi_i + p_i(e_1, e_2)(B - V_i) \text{ for } i = 1, 2. \tag{2}$$

[4] If an agent makes a zero bid in an auction, this can be interpreted as not attending the auction. Accordingly, we assume that an agent who bids zero wins with probability zero, even if all other agents also bid zero. In the equilibrium, this outcome will not occur in any case.

Agent i's payoff is

$$r_i = p_i(e_1, e_2)V_i - \varphi_i - e_i \tag{3}$$

if he accepts the contract (φ_i, V_i), and zero otherwise.

We solve the game by backward induction. No decisions are made in stage 3. Consider stage 2. Suppose both buyers have delegated the bidding to their agents. Due to the contract in stage 1, stage 2 is a standard first-price all-pay auction with full information. The two bidders have valuations V_1 and V_2, respectively. For these given valuations, the Nash equilibrium is in mixed strategies and unique with equilibrium cumulative density functions of bids[5]

$$F_1(e_1) = \begin{cases} 1 - \frac{V_1 - e_1}{V_2} \text{ for } e_1 \in [0, V_1] \\ 1 \text{ for } e_1 > V_1 \end{cases} \quad \text{and } F_2(e_2) = \begin{cases} \frac{e_2}{V_1} \text{ for } e_2 \in [0, V_1] \\ 1 \text{ for } e_2 > V_1 \end{cases} \tag{4}$$

if $V_1 \leq V_2$. The subscripts 1 and 2 in (4) are interchanged if $V_1 > V_2$.

The case in which one buyer i or both buyers do not delegate bidding to an agent leads to the same type of auction equilibrium. The only difference is that party i has valuation $V_i = B$ in the auction if buyer b_i has not delegated the bidding.

[5] See Hillman and Riley (1989), Hirshleifer and Riley (1992), p. 379, and Baye, Kovenock and deVries (1996).

3 Delegation

Consider now the contract stage. Buyer b_i offers a contract (φ_i, V_i) that maximizes her payoff subject to the agent's participation constraint. For both the buyer b_i and her agent, for their contract decision the outcome of the delegation game between the other buyer and her agent is taken as given. That is, they anticipate that there will be a bidder from the other party showing up in the auction in stage 2, and this bidder will have some valuation V_j. For the outcome of subgames in stages 2 and 3, the size of V_j is important. However, for both parties it is irrelevant whether the representative of the other party who shows up at the auction is a buyer or her agent.

We can solve the delegation problem for buyer b_i for all possible valuations V_j. Consider first the up-front fee φ_i. The values of φ_i or φ_j will not affect the outcome of subgames in stages 2 and 3. From (3) and (2) we get that, for given V_i, buyer b_i's payoff increases monotonically in φ_i but agent a_i's payoff decreases monotonically in φ_i. The sum of payoffs does not change in φ_i.

Let p_i^* be the stage-2 equilibrium probability that agent i wins the auction. The agent can anticipate the equilibrium expected return from selling the good to his principal, $V_i p_i^*$ and he anticipates his expected bidding cost, $E(e_i^*)$. By (4), both p_i^* and $E(e_i^*)$ are functions of the valutions V_1 and V_2, but not of the up-front payments. The agent's reservation utility was normalized to zero. This yields the agent's participation constraint

$$\varphi_i \leq V_i p_i^* - E(e_i^*) \text{ for } i = 1, 2. \tag{5}$$

The principal will choose φ_i such that the participation constraint (5) just holds. Taking this into account, the payoff of buyer b_i can be rewritten as

$$\pi_i = Bp_i^* - E(e_i^*) \text{ for } i = 1, 2. \tag{6}$$

Note that buyer b_i's payoff is the same if she does not delegate or if she delegates the bidding and chooses the contract with $V_i = B$ and the according up-front fee that makes (5) binding. Therefore we do not have to distinguish between a buyer who delegates and chooses $V_i = B$ and a buyer who does not delegate, and can treat a buyer who does not delegate like a buyer who has chosen a delegation contract with $V_i = B$.

Now we can state the main result regarding the equilibrium choices of delegated valuations V_i^*. We consider only equilibrium delegation choices in pure strategies.

Proposition 1 *Suppose delegation contracts (φ_i, V_i) with an up-front fee $\varphi_i \geq 0$ and $V_i \in [0, \bar{V}]$ are feasible. (i) If $\bar{V} > B/2$ then exactly two pure strategy equilibrium pairs of delegated valuations exist:*

$$(V_1^*, V_2^*) = (\bar{V}, B/2) \tag{P1}$$

and

$$(V_1^*, V_2^*) = (B/2, \bar{V}). \tag{P2}$$

(ii) If $\bar{V} \leq B/2$ then, if both buyers delegate, the equilibrium delegated valuations are unique:

$$(V_1^*, V_2^*) = (\bar{V}, \bar{V}). \tag{P3}$$

Proof: We calculate buyer b_2's reaction correspondence $R_2(V_1)$. We use that p_2^* in (6) equals $(1-\frac{V_1}{2V_2})$ and $E(e_2^*) = V_1/2$ if $V_1 \leq V_2$, and $p_2^* = V_2/(2V_1)$ and $E(e_2^*) = (V_2)^2/(2V_1)$ otherwise. For each given V_1 buyer 2 therefore maximizes

$$\pi_2(V_1, V_2) = \begin{cases} B(1-\frac{V_1}{2V_2}) - \frac{V_1}{2} & \text{for} \quad V_1 \leq V_2 \\ \frac{BV_2-(V_2)^2}{2V_1} & \text{for} \quad V_1 > V_2, \end{cases} \tag{7}$$

which is a continuous function even at $V_1 = V_2$. If $V_1 \leq B/2$ this function is non-decreasing in V_2 and has a global maximum at $V_2 = \bar{V}$. If $V_1 > B/2$ the payoff function π_2 has two local optima, $V_2 = B/2$ and $V_2 = \bar{V}$.

(i) If $\bar{V} > B/2$ both $V_2 = B/2$ and $V_2 = \bar{V}$ are feasible. Making use of (7), it turns out that $\pi_2(V_1, B/2) > \pi_2(V_1, \bar{V})$ if and only if $V_1 > \hat{V}$ with $\hat{V}$ the larger of the solutions of $\frac{B^2}{8\hat{V}} = B - \frac{B\hat{V}}{2\bar{V}} - \frac{\hat{V}}{2}$. For this solution, $B/2 \leq \hat{V} \leq \bar{V}$ for $\bar{V} \geq B/2$, as the other solution lies outside the interval $[B/2, \bar{V}]$. Therefore, buyer 2's reaction correspondence is

$$R_2(V_1) = \begin{cases} \{\bar{V}\} & \text{for} \quad V_1 < \hat{V} \\ \{(B/2), \bar{V}\} & \text{for} \quad V_1 = \hat{V} \\ \{B/2\} & \text{for} \quad V_1 > \hat{V}, \end{cases} \tag{8}$$

and analogously buyer 1's reaction correspondence $R_1(V_2)$ can be obtained by interchanging subscripts 1 and 2. The delegated valuations (P1) and (P2) in Proposition 1 are the only points of intersection of these reaction correspondences.

(ii) If $\bar{V} \leq B/2$ there remains a single best response $V_2 = \bar{V}$ independent of her opponent's choice. Therefore, in that case the equilibrium in both buyers' strategies is characterized by $V_1^* = V_2^* = \bar{V}$. ■

According to Proposition 1 the two buyers write different contracts with their agents in the more interesting case (i). As a result the actual bidders in the first-price all-pay auction have different delegated valuations, although the actual buyers are perfectly symmetric. The sum of expected payments by the two bidders is lower than in a situation in which the buyers do not delegate bidding to their agents and make bids according to their true willingness to pay.

The intuitive reason for the result is as follows. Suppose buyer 1 chooses some delegated valuation $V_1 > B$. If buyer 2 chooses the same delegated valuation as buyer 1, the agents will not pay any positive fee for the contract, because the agents will bid very competitively and together they will fully dissipate the compensation that the winner receives from his principal. Hence, given that buyer 1 chooses $V_1 > B$ it does not make sense for buyer 2 to choose a high valuation. Both buyers would even make losses in expectation in this case. However, even if $V_1 > B$, buyer 2 can still get some positive payoff. For instance, if buyer 2 chooses $V_2 = B/2$ and $\varphi_2 = 0$, then her agent is willing to accept the contract. Moreover, even though $V_2 < V_1$, by the nature of the mixed-strategy equilibrium, the agent a_2 wins with positive probabilility and sells the good to his principal for $V_2 = B/2$ in case of winning. Accordingly, buyer 2's payoff is at least as high as $[B - (B/2)]$ times the strictly positive equilibrium win probability of her agent. In turn, given that buyer 2 chooses $V_2 = B/2$, buyer 1 can choose a high delegated valuation and still have a positive payoff. Buyer 1's agent will be willing to pay a high fee because the bidding with the other agent will not be very competitive. It turns out in this case that buyer 1 would like to choose the maximum delegated valuation that is feasible.

Corollary 1 *(i) If $\bar{V} > B/2$ the sum of expected bids in the first-price all-pay auction with delegated valuations is equal to $B/4 + B^2/(8\bar{V}) < B$. The expected payoff of the buyer who chooses $V_i = B/2$ equals $B^2/(8\bar{V})$. The expected payoff of the buyer who chooses $V_i = \bar{V}$ equals $3B/4 - B^2/(4\bar{V})$. (ii) If $\bar{V} \leq B/2$ the expected payoff is $(B - \bar{V})/2$ for each buyer and the sum of expected bids is equal to $\bar{V}$.*

Corollary 1 follows from the equilibrium values of delegated valuations in Proposition 1, and the resulting equilibrium cumulative density functions in (4). Without delegation, in the symmetric fully discriminatory first-price all-pay auction the expected payoff of each buyer in the equilibrium is zero. With delegation both buyers obtain a strictly positive expected payoff. The buyer who chooses a high delegated valuation obtains a higher payoff than her competitor if $\bar{V} > B/2$, but payoffs are strictly positive for both buyers.

At this point we should emphasize that this outcome is not the result of collusive behavior. In auctions, collusion between bidders, for instance, agreements about making only very low bids and side payments, can typically increase bidders' joint payoffs. The delegation mechanism considered here, however, is strictly non-cooperative. No contract or binding agreement between buyers is required to end up in the delegation equilibrium.[6]

[6] Not surprisingly the delegation contracts between buyers and their agents here are not re-negotiation proof in the interim stage. Interim renegotiation proofness requires that the equilibrium contracts, e.g. $(\bar{V}, B/2)$, have the property that, once these delegation

We should also note that the gains from delegation are not simply the outcome of co-ordination on one of the two asymmetric pure-strategy equilibria. Consider $\bar{V} > B/2$. A mixed strategy delegation equilibrium would have a delegation stage in which mixed delegation strategies are distributions of delegated valuations. Each buyer chooses the actual delegated valuation randomly according to this distribution. The actual delegated valuation is observed by everyone before the agents enter the all-pay auction. The mixed strategy delegation equilibria are difficult to characterize explicitly. However, there is a simple reason explaining why buyers' payoffs in these mixed strategy equilibria must be strictly positive – whereas payoffs are zero in the equilibrium without delegation. The reason is related to the intuition for Proposition 1: each buyer has a strategy which yields strictly positive payoff, regardless what the other buyer does. Suppose, e.g., buyer 1 chooses a contract with $V_1 = B/2$ and $\varphi_1 = 0$. Whatever the other buyer does, this contract has non-negative payoff for 1's agent: a_1's payoff is zero if $V_2 \geq V_1$, and even strictly positive if $V_2 < V_1$. This makes sure that the agent is willing to accept this contract. Buyer 1's payoff with this contract is

$$\pi_2(V_1, B/2) = \begin{cases} B - \frac{3}{2}V_1 & \text{for} \quad V_1 \leq B/2 \\ \frac{B^2}{8V_1} & \text{for} \quad V_1 > B/2 \end{cases}$$

contracts are signed, there is no scope for secret re-negotiations between the contracting persons. As in most delegation games, this is also not the case here. In a repeated situation, reputation considerations may eliminate this problem. Also, renegotiation may require that the principal and the agent have to meet in the interim stage which may be excluded by technological constraints.

and hence, $\pi_2(V_1, B/2) > 0$ for all $V_1 \leq \bar{V} < \infty$. Intuitively, buyer 1 can make positive payoff even if buyer 2 chooses $V_2 = \bar{V}$, and buyer 1's payoff from this strategy is even higher if buyer 2 may choose a lower delegated valuation. Of course, the same argument applies for buyer 2. Any equilibrium strategy in a mixed strategy equilibrium must therefore yield payoff to each buyer that is not smaller than $\frac{B^2}{8\bar{V}}$.

4 Delegation without fees

The principal-agent contract in section 3 specifies an up-front payment from the agent to the principal and the price for which the principal is commited to purchase the good from the agent. The agent essentially buys an option: the right to sell the good to his principal for a preannounced price in case he wins the auction. In this section we restrict the set of feasible contracts and require that $\varphi_i \equiv 0$. Accordingly, buyers ask their agents to go to a first-price all-pay auction and bid for a good and commit to a price $V_i > 0$ they are willing to pay for the good in case their agent obtains the good. However, the institutional arrangement does not allow agents to make positive payments for these options. A possible reason for this type of restrictions could be credit market constraints.

In section 3 we showed that delegation is worthwhile if the buyers have all bargaining power when designing the delegation contract. In this section we will consider whether this result is robust even if the set of feasible contracts is constrained to contracts without an up-front fee. The principal's expected payoff becomes

$$\pi_i = (B - V_i)p_i^* \text{ for } i = 1, 2, \tag{9}$$

with p_i^* the equilibrium win probability that results for bidders' valuations V_1 and V_2. This win probability and the stage-2 equilibria are the same as in section 3 where we already observed that these subgames are unaffected by the size of up-front fees.

If up-front fees are not feasible, the decision whether a buyer would like to delegate or not, is not obvious. In contrast to section 3, the decision not to delegate is not equivalent to the choice of $V_i = B$. Given that both buyers are commited to delegate, the following proposition characterizes the equilibrium contract.

Proposition 2 *If both buyers delegate and cannot charge an up-front fee, for* $\bar{V} \geq B/2$ *there exists a unique symmetric pair of pure equilibrium delegations:*

$$(V_1^*, V_2^*) = (B/2, B/2). \tag{P4}$$

Proof: Suppose both buyers delegate. We calculate buyer b_2's reaction curve $R_2(V_1)$ that maps anticipated choices of delegation values V_1 of buyer b_1 to the delegated valuation V_2 that maximizes b_2's payoff given V_1.[7] For each given V_1 buyer b_2 maximizes

$$\pi_2(V_1, V_2) = \begin{cases} (B - V_2)(1 - \frac{V_1}{2V_2}) \text{ for } V_1 \leq V_2 \\ \frac{BV_2 - V_2^2}{2V_1} \text{ for } V_1 > V_2, \end{cases} \tag{10}$$

[7] Note that V_1 and V_2 are strictly positive. Otherwise, agents are not willing to make positive bids, and thus, the buyer who chooses a delegated valuation of zero cannot win the auction.

where we have substituted for p_i^* the equilibrium values from the auction game in stage 2. This yields two local optima and the best response is

$$R_2(V_1) = \begin{cases} B/2 \text{ for } V_1 > B/2 \\ \sqrt{BV_1/2} \text{ for } V_1 \leq B/2. \end{cases} \tag{11}$$

For buyer b_1 subscripts 1 and 2 have to be interchanged. Accordingly, the pair (P4) in Proposition 2 is the only point of intersection of these reaction curves. ■

If delegation contracts cannot specify positive up-front fees, it seems possible that agents earn a positive rent in a delegation equilibrium. Proposition 2 shows that this is not the case if both buyers choose to hire their agent. Both buyers give their agent the same delegated valuation, implying that both agents dissipate their rents. Hence, for the agents, the payoffs in the game with zero up-front payments is the same as in the case considered in section 3. However, the delegated valuations in equilibrium (P4) and the principals' payoffs in the game with zero up-front fees differ from their equilibrium payoffs in section 3.

If the delegation contract can specify positive up-front fees, delegation is a (sometimes weakly) dominant strategy. Whether buyers want to delegate if the delegation contracts cannot specify a positive up-front fee will be analyzed now. Suppose each buyer makes a decision whether to hire an agent in a stage prior to the actual delegation stage. This leads to four possible situations, DD, DN, ND and NN, where DD is the game situation in which both buyers hire an agent, in DN buyer 1 hires an agent and buyer 2 does not, etc. We consider the outcomes in these games one after the other.

Consider first DD. The equilibrium delegation contracts are characterized in Proposition 2 for this case. Equilibrium (P4) yields both buyers a payoff equal to $B/4$. The sum of expected payments in the all-pay auction equals $B/2$. The agents in the auction do not earn a rent because they have symmetric delegated valuations and each makes expected bids equal to one half of their valuations.

Consider next NN. If both buyers do not delegate, the buyers' expected payoffs are zero in the equilibrium. Since both buyers have the same valuation of the prize, their expected bids fully dissipate their expected gains from bidding.

Consider now DN (or ND) in which one buyer (say, buyer b_1) hires an agent and delegates bidding to this agent, and the other buyer (buyer b_2) does not hire an agent. The reaction function (11) shows that the buyer who delegates chooses her best response to the given anticipated valuation $V_2 = B$ of party 2. This yields $V_1^* = B/2$. The buyers' equilibrium payoffs are $B/8$ for the buyer who delegates, and $B/2$ for the buyer who does not delegate. Here the seller earns $3B/8$ as the sum of the expected bids.

The decision whether to delegate or not when delegation means writing a contract with zero up-front fees, can be analysed in a two by two matrix, using the payoffs for the four subgames DD, DN, ND and NN.

$1\backslash 2$	N	D
N	$(0,0)$	$(B/2,B/8)$
D	$(B/8,B/2)$	$(B/4,B/4)$

Table 1: Buyers' payoffs depending on their choices to delegate.

Proposition 3 *If both buyers cannot charge an up-front fee, exactly two asymmetric pairs of pure equilibrium delegations exist:* (DN) *with payoffs* $((B/8),(B/2))$*, and* (ND)*, with payoffs* $((B/2),(B/8))$. *The buyer who delegates chooses a delegated valuation* $V_i = B/2$.

The proof of Proposition 3 follows straightforwardly from Table 1 and from the fact that the optimal delegated valuation of i is $B/2$ if the the valuation of the other bidder is B. Hence, there are two asymmetric delegation equilibria, DN and ND. Buyers have strictly positive payoffs, and agents have zero payoffs in these equilibria.

5 Conclusion

In this paper we have shown that it is individually rational for a buyer in a first-price all-pay auction to send an agent who has been given incentives to bid that differ from the incentives of the actual buyer. Non-cooperative delegation choices yield auction equilibria that make all buyers better off than if they themselves make bids in the auction. We considered two types of delegation contracts. In one type of contract the agent pays a strictly positive fee and obtains the right to sell the auctioned good to his principal if he wins the good in the auction. The other type of contract limits the possible fees to zero. It turned out that the buyers' benefits from delegation may be even greater if up-front fees are ruled out.

6 References

Baik KH, Kim IG. 1997. Delegation in contests. *European Journal of Political Economy* **13**: 281-298.

Baik KH, Shogren JF. 1992. Strategic behavior in contest, comment. *American Economic Review* **82**: 359-362.

Baye MR, Kovenock D, deVries CG. 1996. The all-pay auction with complete information. *Economic Theory* **8**: 291-305.

Baye MR, Kovenock D, deVries CG. 1998. A general linear model of contests, mimeo.

Besen SM, Farrell J. 1994. Choosing how to compete: strategies and tactics in standardization. *Journal of Economic Perspectives* **8**: 117-131.

Dixit A. 1987. Strategic behavior in contest. *American Economic Review* **77**: 891-898.

Ellingsen T. 1991. Strategic buyers and the social cost of monopoly. *American Economic Review* **81**: 648-657.

Glazer A, Hassin R. 1988. Optimal contests. *Economic Inquiry* **26**: 133-143.

Hillman A, Riley J. 1989. Politically contestable rents and transfers.

Economics and Politics **1**: 17-40.

Hirshleifer J. 1995. Theorizing about conflict. In *Handbook of Defense Economics* vol. I, Hartley K, Sandler T (eds). North-Holland: Amsterdam; 165-189.

Hirshleifer J, Riley JG. 1992. *The Analytics of Uncertainty and Information*, Cambridge University Press: Cambridge.

Katz E, Nitzan S, Rosenberg J. 1990. Rent-seeking for pure public goods. *Public Choice* **65**: 49-60.

Konrad KA. 2000. Spatial contests. *International Journal of Industrial Organization* **18**: 965-974.

Kräkel M, Sliwka D. 2002. Strategic delegation and mergers in oligopolistic contests. Bonn Econ Discussion Papers DP 2/2002.

Nitzan S. 1994. Modeling rent-seeking contests. *European Journal of Political Economy* **10**: 41-60.

Rosen S. 1986. Prizes and incentives in elimination tournaments. *American Economic Review* **76**: 701-715.

Schelling TC. 1960. *The Strategy of Conflict*, Harvard University Press: Cambridge.

Skaperdas S, Grofman B. 1995. Modeling negative campaigning. *American Political Science Review* **89**: 49-61.

Tullock G. 1980. Efficient rent seeking. In *Toward a Theory of Rent Seeking Society,* Buchanan JM, Tollison RD, Tullock G (eds). Texas A&M Univeristy Press: College Station; 97-112.

Ursprung H. 1990. Public goods, rent dissipation, and candidate competition. *Economics and Politics* **2**: 115-132.

Wärneryd K. 1998. Distributional conflict and jurisdictional organization. *Journal of Public Economics* **69**: 435-450.

Wärneryd K. 2000. In defense of lawyers: moral hazard as an aid to cooperation. *Games and Economic Behavior* **33**: 145-158.