Spatial Contests

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Abstract

Competition in some markets is a contest. Sellers compete for contracts by making promotional efforts. As in an all-pay auction, effort cannot be recovered even if the seller is not awarded the contract. The equilibrium location choices of sellers in a spatial model if firms compete for customers in an all-pay auction are the same as in the standard spatial price discrimination model. However, in contrast to spatial price discrimination, these location choices are inefficient if sellers' competition for customers is a contest.

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1 Introduction

The standard model describing the location choices of sellers if they compete for each single buyer is the spatial price discrimination model by Hoover (1937), Lederer and Hurter (1986) and Hamilton, Thisse and Weskamp (1989). Firms locate at the first stage and make location specific price offers to each customer at a later stage. In some markets, however, uniform delivered prices are fixed due to resale price maintenance or to governmental price regulation.[1] Competing sales representatives earn a commission but are not allowed to sell for a different delivered price. For these markets, price competition is ruled out and sellers contest for contracts with each single customer.

The insurance market in some European countries prior to deregulation in 1994 is an example with strictly enforced resale price maintenance and with location choices of sales representatives with overlapping territories. (For overviews see Finsinger and Pauly 1986, Finsinger, Hammond and Trapp 1985, and Rees and Thimm 1998). In this market, price maintenance was enforced by a governmental regulator. The agents of insurance companies in such markets spend considerable time and money building up a personal relationship with prospective customers, visiting and talking persuasively to them, or sending them promotional literature. In Germany, for instance, insurance companies had to apply for approval by the governmental regulator if they wanted to change the insurance premium on new contracts, or introduce a new type of insurance contract. The approval process was time consuming, representatives of other insurance companies were sometimes involved in the procedure, and a most relevant criterion for approval was whether the insurance premium specified in the contract was sufficiently high to rule out 'ruinous competition'. Effectively, price competition even between insurance companies was almost ruled out. Therefore, even competition between different insurance companies was a contest rather than a standard market game.

Contests also occur in markets for hi-tech products which involve a large informational disadvantage for the buyer, network externalities, and other reasons for a buyer lock-in. If a firm plans to adopt a new software, the decision will crucially depend on expectations about the industry standard in a few years from now. Similarly, the ongoing competition between the aircraft manufacturers in the US and in Europe has strong elements of a contest.[2] In the literature on markets with network externalities (e.g., Besen and Farrell 1994), it has been acknowledged that these markets have characteristics of a contest: sellers make large promotional efforts to

[1] The uniform delivered price may be fixed by a regulator and, hence, cannot be used as a strategic variable by firms here. This avoids the problems of non-existence of equilibrium with endogenous uniform delivered pricing which are well known from Beckmann and Thisse (1986).

[2] Issues like availability of spare parts, expectations about future updates of the manufacturer's product line, and the manufacturer's overall probability of being in business in a few years from now will be more decisive than small price cuts. Producers' provision of credible information on such issues, e.g., expensive investment in capacity (Thum 1995) and other strategic expenditure, is costly and sunk when buyers make their actual purchasing decisions.

persuade buyers that they have the superior product.[3]

In these examples sellers spend resources on promotional product information, money on dining and wining, and time or other forms of effort on persuading a potential buyer that their own product is preferable to others. What distinguishes efforts in such contests from, for example, competing via price reductions is that these efforts are, in effect, sunk costs, which are incurred whether the firm's promotional effort is successful or not, whereas a firm's opportunity cost of selling for a lower price only becomes relevant if the firm is actually awarded the contract.

Sellers must make location choices. For instance, in the price regulated insurance market, individual salesmen representing the same company without exclusive territorial rights make strategic location choices and then contest for customers. Also, competing insurance companies choose the location of their local offices strategically. Similarly, before a producer enters the sales contest, it has to make some basic decisions concerning product design. This decision can be seen as a location choice in the product space. In the context of specialized hi-tech products, there is often a continuum of possible applications for a basic product line. A producer has located somewhere in the product space and has to modify its product and tailor it to the specific needs of a particular customer. These tailoring costs can be regarded as transportation costs and are higher or lower, depending on how distant the producer's location and the customer's location are.

Whereas location choice of firms with price competition with spatial price discrimination has been well studied by Lederer and Hurter (1986) and others, location choice of sellers that compete by contest is far less well understood. The paper characterizes the subgame perfect equilibrium location choice and contest efforts in the equilibrium of this game, compares the outcome with the outcome in the spatial price discrimination model, and considers the efficiency properties of the equilibrium outcome.

2 The Model

Consider the following three stage game. Two firms, A and B, decide simultaneously where to locate on the unit interval. Let the location choices be $a \in [0, 1]$ and $b \in [0, 1]$, respectively. These choices constitute stage 1.

In stage 2 the location $x \in [0, 1]$ of the customer is determined and observed by both firms. It is the realization of a random variable X which is distributed on $[0, 1]$ with a continuous cumulative density function $G(x)$. This customer has inelastic demand of one unit of a good which both firms A and B can supply.

In stage 3 the firms enter a contest: both firms contest for selling a unit to this customer. Firm A spends effort $e_A \geq 0$ to persuade him to contract with firm A,

[3] The pervasiveness of the contest element in allocation processes more generally has been discussed, for instance, by Dixit (1987) and Garfinkel and Skaperdas (1996).

and similarly, firm B uses effort $e_B \geq 0$ to get the contract. Which firm will get the contract depends on which firm expends the greater effort. The probability with which firm A is awarded the contract is

$$p_A(e_A, e_B) = \left\{ \begin{array}{c} 1 \\ 1/2 \\ 0 \end{array} \right\} if \left\{ \begin{array}{c} e_A > e_B \\ e_A = e_B \\ e_A < e_B \end{array} \right\} \tag{1}$$

The probability for B winning is $p_B = 1 - p_A$. Because small differences in effort are decisive, this contest is called a *fully discriminatory* contest, or, using auction language, a first-price all-pay auction.

There are no further decision stages. The winning firm delivers the good. If firm A is located at a and delivers the good to the customer at x, it has transportation cost $c(a, x)$. Cost is a monotonically increasing function of the distance between a and x.[4] Firm B has the same transportation cost function with b replacing a. The customer's payment minus production cost is m. It is exogenous and larger than any possible transportation cost: $m > c(0, 1)$. For simplicity, m is the same for both firms. Depending on the particular market considered, m can be seen, for instance, as the producer's difference between a regulated sales price and production cost, a producer's implicit information rent when awarded the contract, or, applied to the regulated insurance market, the insurance agent's commission.

Firms are risk neutral. If firm A locates in a, B locates in b, the customer is located in x, and sales efforts chosen are e_A, and e_B, then firm A's payoff is obtained as

$$\pi_A = p_A(e_A, e_B)[m - c(a, x)] - e_A, \tag{2}$$

and analogously for firm B.

We solve the game backwards starting with stage 3. For given locations of the firms and the buyer, firm A's and firm B's gains, net of transportation cost, from being awarded the contract are

$$v_A(a, x) \equiv m - c(a, x), \tag{3}$$

and

$$v_B(b, x) \equiv m - c(b, x) \tag{4}$$

respectively. The contest in stage 3 therefore is a symmetric first-price all-pay auction in which the two contestants have valuations of winning a prize, v_A and v_B. It has been shown by Hillman and Riley (1989) and Baye, Kovenock and DeVries (1996) that this contest has a unique equilibrium which is in mixed strategies.[5] This

[4] This cost function is for simplicity. Results can be generalized for a more general cost function as in Lederer and Hurter (1986).

[5] A pure strategy equilibrium can be ruled out by contradiction. Clearly, $(e_A^*, e_B^*) = (0, 0)$ is not an equilibrium. Let the equilibrium be (e_A^*, e_B^*) with some positive effort, e.g., $e_A^* > 0$. Firm B's optimal reply is $e_B^* = 0$ or $e_B^* = e_A^* + \varepsilon$, slightly higher than e_A^*. But this $e_A^* > 0$ is not an optimal reply to $e_B^* = 0$ or to $e_B^* = e_A^* + \varepsilon$. Hence, (e_A^*, e_B^*) is not an equilibrium.

equilibrium is characterized as follows. Let $v_A \geq v_B$. Then the cumulative density functions describing the two firms' mixed bidding strategies are

$$F_A(e_A) = e_A/v_B \quad (5)$$

and

$$F_B(e_B) = 1 - v_B/v_A + e_B/v_A \quad (6)$$

for $0 \leq e \leq v_B$, $F_A(e) = F_B(e) = 0$ for $e < 0$, and $F_A(e) = F_B(e) = 1$ for $e \geq v_B$. Firm B puts positive probability mass on zero effort and distributes the remaining probability mass uniformly on the interval $(0, v_B]$. Firm A puts zero probability mass on zero effort and distributes the whole probability mass uniformly on the same interval $(0, v_B]$. The uniform distribution of probability mass on the support $(0, v_B]$ is an equilibrium requirement that follows from the fact that each firm's probability distribution of efforts must make the other firm just indifferent for all effort choices from its support.[6] The upper limit v_B of the support is based on the following reasoning: firm B will never spend more than $e_B = v_B$ because $e_B > v_B$ implies a certain negative payoff, whereas the payoff is zero if firm B chooses zero effort. Firm A could always choose effort $e_A = v_B$ and obtain a payoff of $v_A - v_B$ with probability 1. This explains that the maximum effort that is observed in the equilibrium is the lower of the two firms' contract valuations. (For $v_B \geq v_A$ all subscripts A and B have to be interchanged in (5), (6) and in the reasoning above.)

Inserting the equilibrium cumulative density functions (5) and (6) into (2) yields

Lemma 1 *The payoffs in the unique equilibrium in the spatial discrimination game with sales contests with a contest success function (1) are*

$$\pi_A = \max\{0, v_A - v_B\} = \max\{0, c(b,x) - c(a,x)\} \quad (7)$$

and

$$\pi_B = \max\{0, v_B - v_A\} = \max\{0, c(a,x) - c(b,x)\}. \quad (8)$$

Let firm B be located further away from the customer, so that $c(b,x) > c(a,x)$. Lemma 1 states for this case that the contest payoff of firm B that has the higher transportation cost is equal to zero. Firm B spends sales effort and wins the customer with some probability. But the expected effort in the equilibrium equals the expected gains from it. This follows from the fact that $e_B = m - c(b,x) = v_B$ is an effort which firm B chooses with positive probability density in the equilibrium. Firm A that is closer to the customer spends more effort, wins with a higher probability and gains more from winning, because it has the lower transportation cost.

[6] Given the mixed strategy equilibrium distribution of firm B's effort, firm A must be indifferent across all equilibrium effort choices $e_A \in (0, v_B)$. If firm A increases its effort by one unit, the marginal cost is one unit. The marginal benefit of this is the gain in firm A's winning probability by $\frac{dF_B(e)}{de}$ at $e = e_A$ times firm A's valuation of the contract. Hence, the marginal condition that must hold in the equilibrium is $1 = \frac{dF_B}{de} v_A$. This explains the uniform distribution $F_B(e_B)$. The same argument applies for firm B and explains uniformity of distribution $F_A(e_A)$.

Overall positive payoff equal to $v_A - v_B$ remains to firm A in the equilibrium. Firm B competes away some rent from firm A. With a contest success function (1) the two firms spend total effort equal to v_B, the rent that the firm with the lower valuation would obtain from winning the contest. The difference $v_A - v_B = c(b,x) - c(a,x)$ is a rent which firm A can get with certainty by choosing e_A slightly above v_B. This rent cannot be contested away in the equilibrium.

It is useful to compare the payoffs (7) and (8) with the payoffs of firms in the standard model of spatial price discrimination with Bertrand competition by Lederer and Hurter (1986). With spatial price discrimination, firms A and B located at a and b compete for a customer located at x. The contract is awarded to firms according to Bertrand competition: the two firms make a price offer simultaneously. The customer accepts the lower price offer. If both firms make the same price offer, Lederer and Hurter assume for simplicity that the customer will accept the offer made by the firm that has the lower transportation cost. As a result, the firm that has the lower marginal cost of delivering to the customer serves this customer at a price equal (or, for different tie breaking rules, slightly below) the marginal cost of the other firm for delivering to this customer. If firms A and B are located at a and b and compete for a customer located at x, their equilibrium payoffs are $\pi_A = \max\{0, c(b,x) - c(a,x)\}$ and $\pi_B = \max\{0, c(a,x) - c(b,x)\}$, precisely the same as in (7) and (8) in the contest game. Therefore, when solving for the subgame perfect location choices in stage 1, the firms' sets of feasible location choices and their objective functions in stage 1 are identical in the spatial discrimination game with sales contests and in the spatial price discrimination game. Hence, both games must have the same location equilibria. This is summarized as

Lemma 2 *The pair of locations (a^*, b^*) is an equilibrium in the location game of firms with spatial price discrimination as in Lederer and Hurter (1986) if, and only if, (a^*, b^*) is an equilibrium in the location game of firms in the spatial discrimination game with sales contests.*

In the Lederer-Hurter spatial price discrimination model with location choices $a \in [0,1]$ and $b \in [0,1]$ and $a \leq b$, the expected transportation costs are

$$\Psi(a,b) \equiv \int_0^{\frac{a+b}{2}} c(a,x)dG + \int_{\frac{a+b}{2}}^1 c(b,x)dG \tag{9}$$

if the customer's location x is chosen at random from the unit interval with a probability distribution described by $G(x)$ and if the customer is served by the firm that is located nearest to him. In this model the equilibria of the location game can be characterized by location choices and market shares that minimize expected transportation cost $\Psi(a,b)$ (see Lederer and Hurter, 1986, Theorem 3). By the equivalence result in Lemma 2, this can be used to characterize location equilibria in the spatial discrimination model with sales contests:

Corollary *If the pair* (a^*, b^*) *of location choices minimizes the expected transportation cost* $\Psi(a, b)$ *for* $(a, b) \in [0, 1] \times [0, 1]$*, then this pair is a subgame perfect location equilibrium in the spatial discrimination game with sales contests.*

This corollary follows straightforwardly from Theorem 3 in Lederer and Hurter (1986) together with Lemma 2 above. However, a direct proof is short and instructive. I show that $a = a^*$ maximizes A's payoff given $b = b^*$. The rest follows from symmetry of the problem. By (7), firm A's objective function is

$$\Phi_A(a, b^*) \equiv \int_0^1 \max\{0, v_A(a, x) - v_B(b^*, x)\} dG, \tag{10}$$

where $v_A(a, x)$ and $v_B(b, x)$ are defined in (3) and (4). This function can be rewritten as

$$\Phi_A(a, b^*) = \int_0^1 c(b^*, x) dG - \Psi(a, b^*), \tag{11}$$

with Ψ defined in (9). Hence, a^* maximizes $\Phi_A(a, b^*)$ if and only if it minimizes $\Psi(a, b^*)$. $\square$

Similar to spatial price discrimination with Bertrand competition, spatial competition with contests renders endogenous incentives for differentiation in the product space. Suppose firm B is located at b. If firm A also locates at b the two firms have the same valuation of selling a contract to a customer at x, for all possible customer locations $x \in [0, 1]$. Firm A's payoff in the contest game is therefore zero. If, instead, firm A locates at some distance from b, some possible customers are now located closer to firm A than to firm B, and firm A has positive payoff in the contest for these customers. Of course, some possible customers are also located closer to B and firm A has zero payoff in the contest for them. Hence, firm A's expected payoff on a randomly drawn customer is positive if A and B choose different locations.

Although spatial price discrimination games and spatial contest games lead to the same location choices, their welfare properties are very different. Consider first welfare for given location choices. In the case of spatial price discrimination, each customer is served by the firm that is located closest to him. Therefore, in the equilibrium transportation cost are minimized and areas in which the competing firms do business are perfectly segregated. Using the sum of rents as a definition of welfare, this implies that, for given location choices, the Bertrand equilibrium in the spatial price discrimination model maximizes welfare.

In contrast, in the spatial contest model each customer is not always served by the firm closest to him. Consider, for instance, customer x and location choices with $x < a < b$. In the spatial price differentiation model, this customer is served by firm A, and transportation costs are equal to $c(a, x)$. In contrast, in spatial contests both firms randomize their contest efforts for a customer according to the unique contest equilibrium characterized in (5) and (6). Both firms serve this customer with strictly positive probability. For $x < a < b$, firm A wins the customer with probability $1 - \frac{1}{2}\frac{v_B}{v_A} = 1 - \frac{1}{2}\frac{m-c(b,x)}{m-c(a,x)}$, and firm B wins the contract with probability $\frac{1}{2}\frac{v_B}{v_A} = \frac{1}{2}\frac{m-c(b,x)}{m-c(a,x)}$. If firm B wins the customer, it has a transportation cost equal to $c(b,x) > c(a,x)$. With spatial contests, a firm delivers to customers in the other firm's hinterland with some positive probability. This implies that, for given location choices, the transportation cost is not minimized in spatial contests.

Consider next the welfare effects of location choices. In the spatial price discrimination model any price reduction that leads to reduced rents on the firms' side leads to an increase in customer rents by the same amount. Transportation cost is a sufficient measure for welfare in this model. In contrast, customers may not gain from increased firms' contest efforts. Contest effort can be pure waste, as is often assumed in the rent-seeking literature. Taking this view, welfare can be described as a function of location choices in the spatial contest equilibrium as

$$\begin{aligned}\Omega(a,b) \quad &\equiv \quad \Gamma + m - \int_0^1 [p_A^*(x,a,b)c(a,x) + p_B^*(x,a,b)c(b,x)]dG \qquad (12)\\ &\quad - \int_0^1 [Ee_A^*(x,a,b) + Ee_B^*(x,a,b)]dG \\ &\quad = \Gamma + \Phi_A(a,b) + \Phi_B(a,b).\end{aligned}$$

The last equality holds by the definition of Φ in (10). In (12), Γ is the customer's rent which is exogenous since in equilibrium the customer receives the good and pays an exogenously given price; $Ee_i^*(x,a,b)$ is the expected equilibrium effort of firm i $(= A, B)$ if the customer is located at x and firms are located at a and b, and $p_i^*(x,a,b)$ is the probability that firm i $(= A, B)$ wins the customer at x in the contest equilibrium for given location choices a and b. This leads to the following

Proposition *Let the pair (a^*, b^*) of location choices be an equilibrium, with $a^* < b^*$. Welfare (12) could be increased if firms locate further away from each other than in (a^*, b^*).*

A proof is as follows. $\frac{\partial \Omega}{\partial a} = \frac{\partial \Phi_A(a^*,b^*)}{\partial a} + \frac{\partial \Phi_B(a^*,b^*)}{\partial a}$. As a^* maximizes $\Phi_A(a, b^*)$ for given b^*, the first term $\frac{\partial \Phi_A(a^*,b^*)}{\partial a} \leq 0$. Further, $\frac{\partial \Phi_B(a^*,b^*)}{\partial a} \underset{(10)}{=} \int_{\frac{a+b}{2}}^1 \frac{\partial c(a,x)}{\partial a} dG < 0$. The argument can be generalized for non-differentiable Φ. $\square$

Proposition 1 states that equilibrium location choices are inefficient. Given that the contracts with customers are allocated in contests, the sum of costs could be reduced if firms located further away from each other.

3 Conclusions

This paper analysed the location choice of firms with firms contesting for customers. It turns out that a subgame perfect equilibrium exists in the location game. The equilibrium in spatial contests has precisely the same location choices as in the spatial price discrimination game. In contrast to the spatial price discrimination model, the equilibrium in the spatial contest model does not minimize expected transportation cost. In particular, the contest model can explain the fact that competing firms' sales districts are not strictly segregated, and why a firm may sell successfully even to customers in the hinterland of its competitor.

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