Abstract. Multinational corporations can shift income into low-tax countries through transfer pricing and debt financing. While developed countries use thin capitalization rules to limit the extent to which a subsidiary can be financed with debt finance, developing countries tend not to do so. In this paper, we analyze the effect on FDI and host country welfare of thin capitalization rules when multinationals can also shift income via transfer prices. We show that while permissive thin capitalization limits may be needed to attract FDI, in developing countries this necessary amount of debt financing facilitates more aggressive transfer pricing and results in lower host country welfare.
1. Introduction.

Concern among countries that they are losing substantial corporate tax revenues because of tax base erosion has been increasing in recent years. The OECD BEPS (OECD, 2013) project points to transfer pricing as the main culprit of this base erosion. Several high-profile studies and newspaper articles also show that the tax planning practices of some multinational companies have become more aggressive over time, raising serious compliance and fairness issues.¹ In particular, if the corporate tax ends up being borne mainly by local firms, these practices have consequences for ownership structures, competition in markets, and the ability to tax capital in the future. Although the concerns over base erosion are well founded, the tax competition literature has provided arguments pointing to some beneficial effects of tax-base competition. For example, Desai, Foley and Hines (2006) argue that while tax planning may reduce revenues of high-tax jurisdictions, it may have offsetting effects on real investment that are attractive to governments. This argument is based on the insight from the tax-competition literature that when capital is perfectly mobile, a source tax on capital falls on immobile factors of production. The reason is that capital outflows following a tax increase lowers worker productivity and thus wages. From a policy point of view, it is therefore better to tax workers directly. Tax planning may help firms avoid the capital tax partly or wholly and thus reduce the adverse effects of inefficient policies.

In a recent paper, Hong and Smart (2010) show that the presence of international tax-planning opportunities created by providing a tax deduction for payments on subsidiary debt allow countries to maintain or even increase high business tax rates, without reducing foreign direct investment. In their model, each multinational firm has a subsidiary located in a tax haven jurisdiction and an operational subsidiary located in a high-tax host country. The multinational firm can invest capital in the host country through a loan from the haven subsidiary so that the tax deductible portion of the interest payments allow the multinational to shift income out of the host country into the low-tax or no-tax haven country. Besides facilitating income shifting out of the host country, the tax deductibility of interest payments also reduces the multinational's after-tax cost of capital and encourages the multinational to increase its overall capital investment in the host-tax country. Increased investment increases the demand for labor which in turn

¹ The case of Starbucks (Reuters, 2012) is a prominent example.
increases the host wage rate and host welfare. The reason is that the high-tax host government somehow is restricted from differentially taxing mobile and immobile tax bases and the tax planning afforded by debt financing helps to remedy this inefficiency. Slemrod and Wilson (2009) show that the net welfare advantage to debt financing disappears when the host country can charge domestic investors and foreign investors different tax rates. One way a host country could do this is to levy a withholding tax on outbound interest payments. However, Johannesen (2012) shows that tax competition among several host countries will, in equilibrium, eliminate any incentive for a host country to impose a withholding tax on interest payments.

Host countries can enact thin capitalization limits to manage or limit the amount of income a multinational shifts into tax havens by limiting the tax deductibility of interest payments. In practice this done by allowing a multinational to deduct interest payments to affiliates only if its debt to equity ratio is not too large. Table 1 lists the basic thin capitalization policies for 54 countries based on information from Blouin et al (2012). Half of the countries have explicit thin capitalization including the largest national economies. Hong and Smart focus on the interaction between a host country's corporate income tax rate and its thin capitalization limit. Their main result shows that host welfare is always increasing in its thin capitalization limit which implies that it is always in a host country's interest to allow (actually promote) the tax planning strategy of debt financing.

In this paper we revisit the analysis in Hong and Smart (2010) by allowing a multinational firm to shift income with transfer prices as well as debt financing. Transfer pricing incentives will be moderated through host country auditing while the amount of interest payments that are tax deductible are limited by what are called thin capitalization rules. While many tax haven papers focus on the role such countries play with regard to debt financing, the reality is that transfer pricing is still considered the major cause of income shifting, and hence of base erosion. Thus, the focus of this paper is on the interaction of transfer pricing behavior and debt financing with a host country's tax rate, where a host country may limit the amount of debt financing with a thin capitalization rule. Our analysis will show that both the amount of debt-financed equity and a multinational's transfer price on the debt together determine the total amount of income a multinational will shift into a tax haven country.

Two issues that did not arise in the Hong and Smart paper emerge when transfer pricing is introduced. First, if a host country's thin capitalization rule allows sufficiently high debt to
equity ratios and the host tax rate is large enough, the multinational's after-tax cost of capital will be negative and the multinational's profit will be unbounded. Sufficiently permissive host policies effectively create a money pump for multinationals to extract host country wealth and undermine host welfare. Second, if the thin capitalization rule does not allow enough debt financing, the host country may not be able to attract any foreign direct investment (FDI). In this case, the host country may need to allow a significant amount of debt financing in order to attract even a small amount of FDI. We show that if the host country's transfer price regulations are weak enough, then adopting this minimal thin capitalization rule may be so large that any strictly positive level of FDI generates less host welfare than zero FDI. It turns out that the constellation of productivity and transfer price regulation parameters that imply that the host country will be worse off adopting permissive tax policies to attract FDI are typical of developing countries. Thus, without strong enough transfer price regulation, a host country will be better off choosing a thin capitalization rule which results in zero FDI.

Section 2 presents the model we analyze. In section 3, we derive conditions on host country tax policies that admit either an equilibrium with zero FDI or one with a strictly positive level of FDI. In section 4, we study optimal host country tax policy. Concluding remarks are offered in section 5.

2. A model of profit-shifting via debt and transfer prices.

We analyze a modified version of the Hong and Smart (2010) (HS) model of a multinational firm. A multinational firm can invest capital, either equity or debt, in a single host country. All of the FDI is issued by a subsidiary of the multinational located in a tax haven country. The sole difference between our model and the HS model is that we allow the multinational to set the transfer price of any capital it lends the subsidiary.

The host country economy consists of workers, who inelastically supply one unit of labor, and entrepreneurs, who own domestic firms. Domestic firms can employ \( L_d \) units of labor at a wage rate \( w \) to produce \( G(L_d) \) units of output that are sold in a competitive market. The output price is normalized to one. The production function, \( G(\cdot) \), is strictly increasing and strictly concave in \( L_d \) and \( G_{L_d} \) is strictly bounded away from 0 and \(-\infty\). The host country levies a profit tax of \( t \) so that the after-tax profit of a domestic firm is
\[ \pi = (1-t)(G(L_d) - wL_d). \]  

(2.1)  

The multinational firm operates with the production function, \( F(L_m, K) \), where \( L_m \) denotes the amount of host-country labor it employs and \( K \) denotes the amount of capital invested in its host country subsidiary. \( F(\cdot, \cdot) \) is strictly increasing, strictly concave, and homogeneous of degree 1 in both inputs. It's second derivatives, \( F_{L_L} \) and \( F_{K_K} \), are also strictly bounded away from 0 and \(-\infty\). The multinational faces the same competitive wage rate, \( w \), and sells its output in a competitive market whose price is also normalized to one. Denote the multinational's economic cost of capital by \( r \).

The multinational can choose to finance its capital investment with equity, \( E \), and/or debt, \( B \), so that \( K = E + B \). We assume that the multinational's economic cost of capital reflects, in part, a country-specific risk of the investment so that \( r \) need not simply equal a worldwide interest rate. The idiosyncratic cost of capital allows the multinational to charge its host country subsidiary an interest rate, \( \sigma \), that can differ from \( r \). That is, \( \sigma \) is the transfer price of debt. Allowing the multinational to use its transfer price on debt to shift income out of the host country is the simplest and most direct way to see the linkages between debt-shifting and transfer pricing.

The multinational incurs transfer pricing costs of \( C(\sigma - r; B; \alpha) = \alpha c(\sigma - r)B \) to reflect any transfer price auditing the host country may conduct. These transfer pricing costs consist of three components. First, the cost function, \( c(\cdot) \), is increasing and convex in the difference between \( \sigma \) and \( r \) which we take to be the arm's-length interest rate. Second, the multinational's transfer pricing costs are proportional to the amount of debt as the total amount of shifted profit will equal \((\sigma - r)B \). Third, we use the non-negative parameter, \( \alpha \), to capture different levels of transfer price auditing intensity by the host country.

A key reason for financing a subsidiary with debt instead of equity is that interest payments on debt are tax deductible expenses while dividend payments to equity holders are not.\(^2\) As long as the subsidiary faces the same tax rate on host country profit, the multinational's after-tax profit is defined as

\(^2\)Davies and Gresik (2003) study the role of debt borrowed from host country investors.
\[ \Pi = (1-t)(F(L_m,K) - wL_m - \sigma B) + \sigma B - rK - C(\sigma - r, B; \alpha). \]  

(2.2)

Many countries impose thin capitalization requirements on multinationals to prevent them from financing foreign operations entirely with debt. As in HS we model thin capitalization rules as the maximum proportion, \( b \), of a multinational's capital investment for which interest expenses can be tax deductible. Assuming that the multinational would use the maximum amount of debt allowed by the host country, \( B = bK \), after-tax multinational profit can be written as

\[
\Pi = (1-t)(F(L_m,K) - wL_m - \sigma bK + \frac{\sigma b - r}{1-t}K) - \alpha c(\sigma - r)bK \\
= (1-t)(F(L_m,K) - wL_m - \rho K)
\]  

(2.3)

where \( \rho = (r - \sigma bt + abc(\sigma - r)) / (1 - t) \) is the effective cost of capital. Because \( C \) is linear in \( B (= bK) \), \( \Pi \) is globally concave in \( (L_m, K, \sigma) \).

The host country seeks to maximize a weighted sum of worker and entrepreneur consumption, \( C_w + \beta C_K \), where \( \beta \in [0,1] \). Aggregate worker consumption equals wage income, \( w \), plus taxes, \( T \). Since \( \rho \) is independent of \( K \), if the multinational elects to invest in the host country it will employ labor and capital so that \( F_L = w \) and \( F_K = \rho \). Thus, host tax revenues equal

\[
T = t\pi + t(F - wL_m - \sigma bK) = t\pi + t(F_K K + F_L L_m - wL_m - \sigma bK) \\
= t\pi + t(\rho - \sigma b)K.
\]  

(2.4)

If one defines host country income as \( Y = F - rK - \alpha cbK + G \), then

\[
Y = wL_m + \rho K - rK - \alpha cbK + \pi + wL_d = (\rho - r - \alpha cb)K + w + \pi.
\]  

(2.5)

Using (2.4), worker consumption is \( C_w = w + T = w + t\pi + t(\rho - \sigma b)K \). Note that
\[ Y - C_w = (1-t)\pi + K((1-t)(\rho - r - \alpha c b + \sigma b t)) = (1-t)\pi = C_e \] where the second equality follows from the definition of \( \rho \). So as in HS, maximizing \( C_w + \beta C_e \) is equivalent to maximizing 
\[ \Omega = Y - (1-\beta)(1-t)\pi . \]

3. No-FDI and positive-FDI equilibria.

There are two possible types of equilibria: no-FDI equilibria and positive-FDI equilibria. The first type of equilibrium occurs when \( K = 0 \) and the second occurs when \( K > 0 \). The first type of equilibria was not considered in HS but will turn out to be important in our model due to the combination of debt shifting and transfer pricing.

First-order conditions for a positive-FDI equilibrium are

\begin{align*}
(i) & \quad F_K = \rho, \\
(ii) & \quad F_L = w, \\
(iii) & \quad G_L = w, \text{ and} \\
(iv) & \quad \alpha c'(\sigma - r)bK = tbK.
\end{align*}

Eq. (3.1.iv) implies that the profit-maximizing transfer price solves

\[ c'(\sigma - r) = t / \alpha \] for any \( K > 0 \) and \( b > 0 \). Denote the solution to (3.2) by \( \sigma^*(t, \alpha) \). The multinational's transfer price is decreasing in \( \alpha \) for \( t > 0 \) (\( \sigma_a^* = -t / (\alpha^2 c''') < 0 \)), independent of \( b \), and increasing in \( t \) (\( \sigma_t^* = 1 / (\alpha c''') > 0 \)). Define the multinational's indirect cost of capital, \( \rho^*(b, t, \alpha) \), as \( \rho \) evaluated at \( \sigma^* \). Eq. (3.2) implies that

\[ \rho^*(b, t, \alpha) = \frac{r(1-bt)}{1-t} - \frac{b[t(c')^{-1}(t / \alpha) - \alpha c((c')^{-1}(t / \alpha))]}{1-t} \]
so that with the strict convexity of \( c(\cdot) \), \( \rho^* \leq \frac{r(1-bt)}{(1-t)} \) and \( \sigma^* t \geq \alpha c(\sigma^* - r) \). The multinational's indirect cost of capital is increasing in \( \alpha \) for \( b > 0 \) and \( t > 0 \) (\( \rho^* = \frac{cb}{(1-t)} > 0 \)), decreasing in \( b \) for \( t > 0 \) (\( \rho^*_b = -\frac{\sigma^* t - \alpha c}{(1-t)} < 0 \)), and can be increasing or decreasing in \( t \) (\( \rho^*_t = \frac{(\rho^* - \sigma^* b)}{(1-t)} \)). For \( b \) sufficiently close to zero, \( \rho^* \) will be increasing in \( t \), while for \( t \) and \( b \) sufficiently large, \( \rho^* \) can be decreasing in \( t \) since the multinational can now shift a significant amount of income out of the host country via its transfer price.

Eqs. (3.1) plus the labor market clearing condition, \( L_d + L_m = 1 \), define a positive-FDI equilibrium if they admit a solution with \( K > 0 \). Denote such an equilibrium by \( K(b,t,\alpha) \), \( L_m(b,t,\alpha) \), \( L_d(b,t,\alpha) \), \( \sigma^*(t,\alpha) \), and \( w^*(b,t,\alpha) \) and denote equilibrium host welfare by \( \Omega^*(b,t,\alpha) \).

3.1 Equilibrium existence.

No equilibrium with \( K = 0 \) or \( K > 0 \) will exist if the multinational's profit function is unbounded. For each \( b > 0 \), this situation will arise if \( t \leq 1 \) and \( \rho^* < 0 \) or if \( t < 1 \) and \( \rho^* = 0 \) due to the multinational's ability to set \( \sigma \). This issue does not arise in HS due to the absence of income shifting via transfer prices in their model. To identify the host country policies for which no equilibrium exists note that \( \rho^* = 0 \) only if the numerator of \( \rho^*, r - \sigma^* bt + \alpha cb \), equals zero. For \( \alpha \) fixed, \( b \) and \( t \) imply \( \rho^* = 0 \) if, and only if, \( b = \frac{r}{(\sigma^* t - \alpha c(\sigma^* - r))} \). If for some \( t \) this value of \( b \) is greater than one, then \( \rho^* \) is always positive. This equation defines an iso-\( \rho^* \) curve in \((b,t)\) space. Denote this curve by \( b_\alpha(t,\alpha) \). Comparative statics calculations imply that \( \partial b_\alpha / \partial t < 0 \) and \( \partial^2 b_\alpha / \partial t^2 > 0 \). The strict convexity of \( c(\cdot) \) also implies that \( 0 < b_\alpha(1,\alpha) < 1 \) and \( b_\alpha(t,\alpha) = 1 \) for \( t \) strictly between 0 and 1.

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3 Define \( x = (c')^{-1}(t / \alpha) \). Then one can write the term in square brackets in (3.3) as \( \alpha(xc'(x) - c(x)) \) which is non-negative by the strict convexity of \( c(\cdot) \).

4 Note that \( \rho^* \) is undefined at \( t = 1 \). In this case, \( Jt = -(r - \sigma b + \alpha cb)K \) so the multinational's profit will continue to be unbounded when \( r - \sigma^* bt + \alpha cb < 0 \). When \( r - \sigma^* b + \alpha cb = 0 \) at \( t = 1 \), L'hôpital's Rule implies that \( \lim_{t \to 1} \rho^* = \sigma^* b \). Otherwise the limit is either \( \infty \) or \( -\infty \).
Proposition 1. For each $\alpha$, no equilibrium exists due to unbounded multinational profit for all $b \geq b_\alpha(t, \alpha)$ when $t \in [0,1)$ and for all $b > b_\alpha(1, \alpha)$.

Proposition 1 indicates that for each $t$ and $\alpha$, sufficiently high values of $b$ will allow the multinational to earn infinite profits through its ability to charge its host country subsidiary a sufficiently high interest rate so that its after-tax cost of capital is negative. Furthermore, this issue cannot be avoided by appealing to Inada-type conditions as it is not the result of production function properties but rather by the multinational's response to the host country's tax policies. In addition at $\rho^* = 0$, $\rho^*_t = (\rho^* - \sigma^* b) / (1 - t)$ is strictly negative for all $b > 0$. Thus, the set of values of $b$ for no equilibrium exists is increasing with $t$. That is, large values of $b$ permit the multinational to shift a substantial amount of income to the tax haven by financing a large proportion of investment via debt while a high tax rate encourages a higher transfer price. The thick solid curve in Figure 1 represents the set of values of $b$ and $t$ for which $\rho^* = 0$.

Higher values of $\alpha$ ameliorate this existence problem by increasing the multinational's indirect cost of capital for each $b > 0$. In the limit as $\alpha \to \infty$, $\rho^* = 0$ only for $b = t = 1$. Thus, when any deviations from the arm's-length interest rate, $r$, are infinitely expensive, an equilibrium exists for all $b$ and $t$ as in HS.

3.2 No-FDI equilibrium.

Another possibility is that the host factor markets equilibrate only when there is no FDI, that is, $K = 0$. This situation can occur when the host wage rate without FDI is too high relative to the multinational's cost of capital. Without any FDI, the equilibrium wage in an equilibrium with zero FDI is $w_0 = G_L(1)$. Because $F$ is homogeneous of degree 1 in both inputs, one can write

$$F(L_m, K) = L_m \cdot F(1, K / L_m) = K \cdot F(L_m / K, 1). \quad (3.4)$$

Eq. (3.4) then implies that $F_K(L_m, K) = F_K(1, K / L_m)$ and $F_L(L_m, K) = F_L(L_m / K, 1)$. Since $F_K$ is monotonic in $K$, (3.1.i) implies that if there exists a positive-FDI equilibrium, then
\[ \frac{K}{L_m} = F^{-1}_K(\rho^*). \] Substituting this expression into (3.1.ii) implies

\[ F_L(1 / F^{-1}_K(\rho^*),1) = w^*. \] (3.5)

Because a positive amount of FDI will increase the demand for host labor, a necessary condition for a positive equilibrium to exist is \( w^* > w_0 \) or

\[ F_L(1 / F^{-1}_K(\rho^*),1) > G_L(1). \]

Denote the value of \( \rho^* \) for which \( w^* = w_0 \) by \( \rho_0 \). The associated iso- \( \rho^* \) curve is a function, \( b_0(t,\alpha) \). Since \( \rho_0^* < 0 \), the only equilibrium is one with no FDI for all \( b \leq b_0(t,\alpha) \). If for some \( t \), a positive-FDI equilibrium exists at \( b = 0 \), then one can define \( b_0(t,\alpha) \) to be strictly negative.

**Proposition 2.** For each \( \alpha \), the multinational invests no capital in the host country in equilibrium for all \( b \leq b_0(t,\alpha) \) when \( t \in [0,1) \) and for all \( b < b_0(1,\alpha) \).

Proposition 2 implies that there is a minimum value of \( b \) needed to support positive FDI in equilibrium as the following example illustrates.

**Example 1.** Assume that \( G(L_d) = L_d^\lambda \) for \( \lambda \in (0,1) \) and \( F(L_m,K) = K^\gamma L_m^{1-\gamma} \) for \( \gamma \in (0,1) \). A positive-FDI equilibrium will exist if, and only if, \( w^* > \lambda \). This condition is equivalent to \( \rho^* < \gamma((1-\gamma) / \lambda)^{(1-\gamma)/\gamma} \). If \( \lambda + \gamma = 1 \), then \( \rho^* \) must be less than \( \gamma \). For \( c(\sigma - r) = \alpha(\sigma - r)^2 \), \( r = .15, t = .3, \lambda = .9, \alpha = .1, \) and \( \gamma = .1 \), positive-FDI equilibria will exist only if \( b > .17 \).

For all \( \rho^* \) that can arise in a positive-FDI equilibrium, differentiating (3.5) with respect to \( \rho^* \) implies

\[ \frac{d w^*}{d \rho^*} = F_L(1 / F^{-1}_K(\rho^*)) \cdot (-1 / F^{-1}_K(\rho^*)^2) \cdot dF^{-1}_K(\rho^*) / d \rho^* < 0 \] (3.6)
because $K / L_m = F_k^{-1}(\rho^*)$ implies that $dF_k^{-1} / d\rho^* = 1 / F_{kk}$. Ineq. (3.6) thus shows that the multinational's indirect cost of capital and the equilibrium host country wage are negatively correlated. Host country policies that lower $\rho^*$ also increase $w^*$ and can increase the multinational's cost of production and discourage FDI. In order for the equilibrium host country wage to support FDI, the multinational's indirect cost of capital cannot be too large.

Note the tension between Propositions 1 and 2. By Proposition 1, if $b$ is too large, then the multinational's profit is unbounded and no equilibrium will exist. By Proposition 2, if $b$ is too small, the only equilibrium may be one with zero FDI. And, increases in $\alpha$ increase the set of values of $b$ and $t$ that imply zero FDI in equilibrium.

This tension is further illustrated in Figure 1 in which several iso-$\rho^*$ curves are graphed. The thickest solid curve is the iso-$\rho^*$ for $\rho^* = 0$. As $\rho^*$ increases, the corresponding iso-$\rho^*$ curve shifts down and to the left, as indicated by the other solid curves. For $\rho^* \leq r$, the iso-$\rho^*$ curves will be downward sloping for all $t$ as when $\rho^* = 0$. For $\rho^* > r$, the iso-$\rho^*$ curves begin at $b = 0$ and are initially increasing in $t$. The lower solid curve (with intermediate thickness) is the iso-$\rho^*$ curve for $\rho_0$. Note that all the iso-$\rho^*$ curves converge at $t = 1$. Above this point of convergence (the open circle) $\rho^* = -\infty$, while below it $\rho^* = \infty$. Footnote 4 reports that, for $b$ fixed, $\rho^*$ converges to $\sigma^* b$. However, any other limiting value can be achieved by converging $b$.
along the appropriate iso-$\rho^*$ curve. As a result, at the point of convergence multiple equilibria exist with $K$ ranging from 0 to $\infty$. For $t < 1$, positive-FDI equilibria will exist only for values of $b$ and $t$ that fall in between the two thick solid lines. The dashed lines correspond to the iso-$\rho^*$ curves for $\rho^* = 0$ and $\rho^* = \rho_0$. They show that fewer combinations of $b$ and $t$ result in no equilibrium existing and more combinations of $b$ and $t$ result in zero FDI in equilibrium.

Corollary 1. If $\rho_0 < r$, then for all $t$ the minimum value of $b$ for which a positive-FDI equilibrium exists is strictly greater than zero. That is, $b_0(t, \alpha) > 0$ for all $t$.

Corollary 1 addresses the case in which the multinational would not invest in the host country without some tax incentive. In this case, the host country would not only need to allow for some tax-preferred debt financing but would also need to set a strictly positive tax rate in order to create an incentive for income shifting via transfer pricing.

4. Host country welfare maximization.

In any positive-FDI equilibrium $K$ and $L_m$ are strictly positive. Thus, totally differentiating the equations in (3.1) and using the fact that $dL_d = -dL_m$ implies when $b > 0$ that

$$
\begin{pmatrix}
F_{KK} & F_{KL} & 0 & 0 \\
F_{KL} & F_{LL} & -1 & 0 \\
0 & -G_{LL} & -1 & 0 \\
0 & 0 & 0 & \alpha c''
\end{pmatrix}
\begin{pmatrix}
dK \\
dL_m \\
dw^* \\
d\sigma^*
\end{pmatrix}
=
\begin{pmatrix}
d\rho^* \\
0 \\
0 \\
dt - c'd\alpha
\end{pmatrix}
$$

where $d\rho^* = \rho^*_0 db + \rho^*_a d\alpha + \rho^*_t dt$. Also define

$$
H = \begin{pmatrix}
F_{KK} & F_{KL} & 0 \\
F_{KL} & F_{LL} & -1 \\
0 & -G_{LL} & -1
\end{pmatrix}
$$
and note that \(|H| = -(F_{kk}F_{LL} - F_{kl}^2) - F_{kk}G_{LL} < 0\). Solving (4.1) then implies

\[
dK = -d \rho^*(F_{LL} + G_{LL})/|H|, \tag{4.2}
\]
\[
dL_m = d \rho^*F_{lk}/|H|, \tag{4.3}
\]
\[
dw^* = d \rho^*F_{lk}/F_{kk}, \tag{4.4}
\]
and
\[
d\sigma^* = (dt - c'd\alpha)/\alpha c''. \tag{4.5}
\]

Then, totally differentiating equilibrium host country welfare implies

\[
d\Omega^* = ((\rho^* - \alpha c)db + \rho^*_{\alpha}dt + (\rho^*_{a} - cb)d\alpha - tbd\sigma^*)K + (\rho^* - r - \alpha cb)dK + (1 - \beta)\pi dt + [1 - (1 - (1 - \beta)(1 - t))L_d]dw^*. \tag{4.6}
\]

Substituting (4.2)-(4.5) into (4.6) yields

\[
d\Omega^* = db \left\{ -\frac{(\sigma^* t - \alpha c)\Gamma}{(1-t)} - \alpha cK \right\} + dt \left\{ \frac{(\rho^* - \sigma^* b)\Gamma}{(1-t)} - \frac{btK}{\alpha c''} + (1 - \beta)\pi \right\} \tag{4.7}
\]
\[
+ d\alpha \left\{ \frac{cb\Gamma}{(1-t)} + \left( \frac{bt^2}{\alpha^2 c''} - cb \right)K \right\}
\]

where

\[
\Gamma = K - (\rho^* - r - \alpha cb)\frac{F_{ll} + G_{ll}}{|H|} + \frac{F_{lk}}{F_{kk}}[1 - (1 - (1 - t)(1 - \beta))L_d]
\]
\[
= K - (\rho^* - r - \alpha cb)\frac{F_{ll} + G_{ll}}{|H|} - \frac{K}{L_m}[1 - (1 - (1 - t)(1 - \beta))L_d] \tag{4.8}
\]
\[
= -(\rho^* - r - \alpha cb)\frac{F_{ll} + G_{ll}}{|H|} - (1 - t)(1 - \beta)KL_d/L_m,
\]

the second line of (4.8) is due to the CRS assumption on \(F\), and the last line of (4.8) is due to the fact that \(L_m + L_d = 1\) in equilibrium. If the marginal welfare return to FDI, \(\rho^* - r - \alpha cb\), is non-negative, then \(\Gamma < 0\). However, if the marginal welfare return to FDI is sufficiently positive to imply that \(\Gamma \geq 0\), then (4.7) implies that host welfare will be decreasing in \(b\).
4.1 A benchmark case: The HS model.

In the HS model, there are no opportunities for transfer pricing. Taking the limit of our model as $\alpha$ goes to infinity approximates the HS model. The multinational's optimal transfer price, $\sigma^*$, converges to $r$ and one can use L'Hopital's Rule to show that $\alpha c$ converges to zero.\(^5\)

\[
p^* - r - \alpha cb \rightarrow rt(1 - b) / (1 - t) > 0 \text{ for all } b < 1 \text{ so } \Gamma < 0 \text{ in the limit.}
\]

Consistent with Proposition 4 in HS, (4.7) implies that for positive-FDI equilibria, $\frac{\partial \Omega^*}{\partial b} = -\alpha t \Gamma / (1 - t) > 0$. Thus, increasing the amount of income the multinational can shift out of the host country via debt financing increases host country welfare for all $b$. An implication of this result not made in HS is that the optimal value of $b$ is one for all $t > 0$. Furthermore, setting $b = 1$ implies that $t$ is now a pure-profit tax for both domestic and foreign firms. Because the tax rate does not distort the firm's factor demands, a positive-FDI equilibrium will exist for every $t$ (with $b = 1$). As a result, $\frac{\partial \Omega^*}{\partial t} = (1 - \beta)\pi$. Thus, the optimal host country policy is one in which the host government allows the multinational to deduct all of its capital costs (financed with debt), appropriates all of the profits from domestic and foreign firms by imposing a tax rate of 100%, and then redistributes the tax revenues it to workers. Equilibrium multinational profit is zero.

The implication of this benchmark analysis is that the ability of the host country to completely shut down the transfer pricing channel creates a strong incentive for the host country to allow full debt-shifting as all the rents from this activity can be completely taxed away.

4.2 Optimal host country policy with debt shifting and transfer pricing.

For any finite value of $\alpha$, the host country policy of setting $b = t = 1$ is no longer optimal. As shown in section 3.1, this policy would give the multinational the means to shift an arbitrarily large amount of profit out of the host country before it is taxed. The quantity demanded on host labor by the multinational would be effectively zero so the welfare gain in the HS model attributable to a higher wage rate will not materialize. If the optimal host policy is to attract zero FDI, the optimal host tax policy is a tax rate of one, since it allows the government to extract all the rents from domestic firms, and any $b$ consistent with zero FDI.

\(^5\) However, even in the limit, $\alpha c' = t$, so a marginal effect of transfer pricing still persists as long as a positive-FDI equilibrium survives in the limit.
Treating $\alpha$ as an exogenous parameter, the host country's first-order conditions with respect to $b$ and $t$ for an interior optimum are

$$\frac{\partial \Omega^*}{\partial b} = -\frac{(\sigma^* t - \alpha c)\Gamma}{(1-t)} - \alpha c K = 0$$  \hspace{1cm} (4.9)$$

and

$$\frac{\partial \Omega^*}{\partial t} = \frac{(r - \sigma^* b + \alpha c b)\Gamma}{(1-t)^2} - \frac{btK}{ac''} + (1-\beta)\pi = 0.$$  \hspace{1cm} (4.10)$$

The first term in (4.10) is the indirect effect of a change in welfare from a change in $t$ transmitted through a change in $K$, $L_m$, $L_d$, and $\rho^*$. The second term is the indirect welfare effect from a change in $t$ generated by a change in the transfer price. The last term is the direct effect of a change in $t$ from the host country's ability to extract more domestic firm rent. In the limit when $t \to 1$, the first term converges to $-\infty$. This means the host country would never want to induce a positive level of FDI with a tax rate of 1. The optimal HS tax rate is thus only optimal if the host country prefers zero FDI.

**Proposition 3.** For all finite $\alpha$, it is never optimal for the host country to set $t = 1$ if it wants the equilibrium level of FDI to be positive.

Given the possibility of an equilibrium with zero FDI, deriving the optimal host country tax policy requires that one compare host welfare under the best no-FDI equilibrium to host welfare under the best positive-FDI equilibrium. To determine the latter, we can start by asking when it is optimal for the host country to allow some tax planning via debt financing. Eq. (4.9) reveals that the host country has a strict preference for setting $b > 0$ to attract FDI only if $\Gamma < 0$ either at $b = 0$ (if $b = 0$ implies that $\rho^* < \rho_0$) or at the value of $b$ for which $\rho^* = \rho_0$. Consider the host country's preferences near the boundary between positive-FDI equilibria and no-FDI equilibria. For any values of $b$ and $t$ that imply $\rho^* = \rho_0$, $K = 0$. If at $\rho^* = \rho_0$, $\Gamma < 0$ then

$$\frac{\partial \Omega^*}{\partial b} > 0.$$
**Proposition 4.** Fix \( t > 0 \). If the limit of \( \Gamma^* \) as \( \rho^* \) converges to \( \rho_0 \) from below is strictly negative, then the host country will prefer a positive-FDI equilibrium to any no-FDI equilibrium.

Note the inherent tension between the multinational's marginal cost of FDI and the host country's marginal welfare return to FDI. According to Proposition 2, the multinational will invest in the host country only if \( b \) is sufficiently large. However, \( \rho^* - r - \alpha cb \) is decreasing in \( b \) and \( \rho^* - r - \alpha cb \geq 0 \) if, and only if, \( b \leq r / (\sigma^* - \alpha) \) so larger values of \( b \) are more likely to imply that host welfare is decreasing in \( b \).

With the host tax rate fixed, Proposition 4 provides a sufficient condition for the host country to prefer an equilibrium in which it allows enough debt financing to attract a positive amount of FDI. A stronger sufficient condition that implies \( \Gamma < 0 \) is \( \rho_0 - r - \alpha c \geq 0 \). In other words, the host country will prefer choosing a strictly positive \( b \) to induce positive FDI if the marginal welfare return to FDI is positive at the effective cost of capital that shuts down FDI. This condition is only sufficient and not necessary. If at \( \rho^* = \rho_0^- , \Gamma \geq 0 \) then \( \partial \Omega^* / \partial b < 0 \) which means the host government will locally prefer a no-FDI equilibrium to positive-FDI equilibria with \( K \) close to zero. Since \( \Omega^* \) need not be globally concave in \( b \), \( \Gamma \geq 0 \) at \( \rho^* = \rho_0^- \) is only a necessary condition for the host country to prefer no-FDI.

An extended analysis of the economy described in Example 1 can help us better understand the conditions under which a host country would prefer zero FDI to a positive level of FDI. For the production functions in Example 1, an increase in \( b \) can never cause the sign of \( \Gamma \) to switch from positive to negative. Thus, if for a fixed \( t, \Gamma \geq 0 \) at \( \rho^* = \rho_0^- \), then the host country prefers a no-FDI equilibrium to any positive-FDI equilibrium holding \( t \) fixed.

Using the same assumptions as in Example 1, Figure 2 shows that it is indeed possible for the host country to prefer an equilibrium with zero FDI to one with a positive amount. In this example, the host country must allow a substantial amount of tax-favored debt financing in order to attract any FDI. As \( b \) increases, the multinational's cost of capital falls as does the host country's welfare return on FDI. If this minimum value of \( b \) is large enough, any FDI that is invested in the marginal welfare return to FDI will be negative so that any FDI will decrease host country welfare. In this example, this negative result arises from a combination of a high
interest rate, very capital intensive foreign production, very labor intensive domestic production, and low transfer price costs; conditions that one might often expect to see in developing countries. Moreover, this example is robust to changes in $t$ as the optimal host policy is $b = 0$ and $t = 1$. With the optimal policy, the host country offers no tax benefit for debt financing, attracts no FDI, and taxes all of the rents of the domestic entrepreneurs.

5. Conclusion.

This analysis demonstrates that allowing multinationals to shift income out of a host country using debt financing and transfer pricing introduces equilibrium behavior not observed in Hong and Smart (2010). First, it is now possible for a multinational's after-tax cost of capital to be negative. The combination of a sufficiently high corporate income tax rate and a sufficiently high limit on tax deductible interest expenses can create a money pump that allows a multinational to shift arbitrarily large amounts of income out of the host country. Second, if the host country does not allow the multinational to deduct enough of its interest expenses on internal debt, the host country will not be able to attract any FDI. The combination of these first two results implies that the optimal tax rate is strictly less than the optimal rate that arises with debt financing as the sole channel for income shifting. Third, if the host country must allow for very thinly capitalized subsidiaries (by setting $b$ high) in order to attract FDI, the host country may be made worse off adopting policies that attract FDI than with policies that attract no FDI.
References
Appendix

**Proposition 1.** For each $\alpha$, no equilibrium exists due to unbounded multinational profit for all $b \geq b_{\alpha}(t, \alpha)$ when $t \in [0,1)$ and for all $b > b_{\alpha}(1, \alpha)$.

Proof of Proposition 1.

For $t < 1$, if $b = b_{\alpha}(t, \alpha) \equiv r / (\sigma^* t - \alpha c)$, then $\rho^* = 0$. Because $\rho_b^* < 0$, $\rho^* \leq 0$ and the multinational's profit is unbounded for all $b \geq b_{\alpha}(t, \alpha)$. For $t = 1$, $\Pi = -(r - \sigma^* b + \alpha c b)K$.

Thus, the multinational's profit is unbounded for all $b > b_{\alpha}(t, \alpha)$. For $b = b_{\alpha}(t, \alpha)$, $\Pi \equiv 0$ so any amount of capital and labor is profit-maximizing.

Given (3.2), $b_{\alpha}(1, \alpha) = r / (\sigma^* - \alpha c) = r / (r + (c')^{-1}(1/\alpha) - \alpha c((c')^{-1}(1/\alpha))) > 0$. By the strict convexity of $c()$, $b_{\alpha}(1, \alpha)$ is also strictly less than one. Next note that as $\partial b_{\alpha} / \partial t < 0$, $b_{\alpha}(t, \alpha) < 1$ for all $t$. Finally note that $b = 1$ and $t = 0$ imply $r = 0$ in order for $\rho^* = 0$. Thus, $b_{\alpha}(t, \alpha) = 1$ for some $t \in (0,1)$.

**Q.E.D.**

**Proposition 3.** For all finite $\alpha$, it is never optimal for the host country to set $t = 1$ if it wants the equilibrium level of FDI to be positive.

Proof of Proposition 3. The proof involves calculating the limit as $t$ converges to 1 of

$$
\lim_{t \to 1} \frac{(r - \sigma^* b + \alpha c b) \Gamma}{(1-t)^2}.
$$

Using the definition of $\Gamma$ in (4.8),

$$
\lim_{t \to 1} \frac{(r - \sigma^* b + \alpha c b) \Gamma}{(1-t)^2} = \lim_{t \to 1} \frac{-(r - \sigma^* b + \alpha c b) (\rho^* - r - \alpha c b) (F_{LL} + G_{LL}) + (r - \sigma^* b + \alpha c b) (1 - \beta) KL_d}{(1-t)^2 |H|}.
$$

Direct calculation shows that for any $t < 1$ that $\rho^* - r - \alpha c b = t (r - \sigma^* b + \alpha c b) / (1-t)$ so

$$
\lim_{t \to 1} \frac{(r - \sigma^* b + \alpha c b) \Gamma}{(1-t)^2} = \lim_{t \to 1} \frac{-t (r - \sigma^* b + \alpha c b)^2 (F_{LL} + G_{LL}) + (r - \sigma^* b + \alpha c b)(1 - \beta) KL_d}{(1-t)^3 |H|}.
$$

\[ (A.1) \]
For any finite $\rho^*, K / L_m$ is finite while $F_{LL}, G_{LL},$ and $|H|$ are all finite and non-zero by assumption. Thus, evaluating the right-hand side of (A.1) at $t = 1$ yields 0/0. Using L'Hopital's Rule twice implies that

$$\lim_{t \to 1} \frac{(r - \alpha^* b + \alpha cb)\Gamma'}{(1-t)^2} = -\frac{(b / \alpha c'')(F_{LL} + G_{LL})}{6(1-t)|H|} = -\infty.$$ (A.2)

Eq. (A.2) in turn implies that $Q'_t \to -\infty$ as $t \to 1$. Thus, among all $(b,t)$ for which $K > 0$ in equilibrium, $t = 1$ cannot be optimal for the host country. $Q.E.D.$
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