Challengers and Electoral Accountability*

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It is often argued that serious challengers are essential to the proper functioning of democracy. For example, Ian Shapiro writes

one of the principal reasons why opposition and political competition are essential to democratic politics is that they provide the mechanism through which democratic leaders are held to account. [The Moral Foundations of Politics, pp. 200-201]

Formal theories of accountability try to capture this idea, modeling elections in which the threat of removal may keep the incumbent in line. A viable challenger is crucial for this to work, because the voter’s threat to remove the incumbent is not credible if the outside option is unacceptable. In almost all existing models, however, the challenger is completely passive, so the models (by construction) overlook key tasks that challengers perform, such as criticizing the incumbent’s performance. The importance of analyzing challengers as active players, rather than passive replacements, is underscored by V.O. Key’s oft-cited passage:

if a legislator is to worry about the attitude of his district, what he needs really to worry about is, not whether his performance pleases the constituency at the moment, but what the response of his constituency will be in the next campaign

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when persons aggrieved by his position attack his record. The constituency, thus, acquires a sanction largely through those political instruments that assure a challenge of the record. In the large, that function is an activity of the minority party. (Public Opinion and American Democracy, p. 499)

Several aspects of Key’s argument are worth highlighting. First, the incumbent is making decisions in a forward-looking way, taking into account what voters will believe on election day as a function of what happens in the meantime. Second, voters’ election day beliefs are affected by attacks made by interested parties, rather than just voters’ direct observations. Third, these challenges of the incumbent’s record are what provide the voter with meaningful possibilities to sanction the incumbent for poor performance. And fourth, the opposition party is the main actor making the challenges.

Although Key’s argument is an attractive account of how democratic contestation might work, it is not obvious that his claims hang together logically. The main problem is that an electoral challenger’s goal is not to provide neutral information about incumbent performance, but rather to win office. This drives a wedge between the interests of the challenger and those of the voters. So the question becomes: will a challenger transmit information that is useful to voters who know he has an incentive to misrepresent or unfairly criticize the incumbent’s record in order to enjoy the benefits of office himself? What makes voters’ problem particularly tricky is that they have a very limited set of tools at their disposal—all that they can do is to retain the incumbent or replace her with the challenger. This one tool must simultaneously induce the incumbent to choose correct policies and induce the challenger to supply information about the incumbent’s record. Moreover, as noted by Fearon (1999), at the time of the election, voters must rationally focus on selecting the next leader rather than sanctioning the current one for bad performance.

In this paper we analyze a political agency model in which an electoral challenger can actively monitor the incumbent’s policymaking. In our model, the incumbent can exert effort to acquire information about what policy choices best promote voters’ interests, and the challenger can gather information about whether the incumbent’s policy choices indeed served their interests.

If the challenger in this model is simply a passive replacement, as in most accountability models, the voter can sometimes provide the incumbent with strong incentives to acquire information proving that she has chosen the correct policy. But this only works when the two candidates are sufficiently close ex ante. If the replacement is well ahead of or well behind the incumbent, the voter will simply retain or replace the incumbent based on his prior beliefs about the two politicians, and thus the incumbent has no incentive to choose
good policies and doesn’t reveal any electorally-relevant information.

An active challenger, who can obtain information about the incumbent’s policy choices, can serve as an effective monitor, in the sense that the voter removes the incumbent whenever the challenger obtains information proving that she made bad policy choices. Such monitoring is quite useful, both for incumbent incentives and for selection, if the incumbent is substantially stronger than the challenger ex ante. Monitoring can also sometimes be useful if the incumbent is substantially weaker than the challenger. However monitoring isn’t always useful; compared to direct accountability based on the incumbent’s own actions and announcements, it is a less-effective tool for giving the incumbent an incentive to exert effort to improve her policymaking. Thus, when the incumbent and challenger are sufficiently close ex ante, voters can actually be better off ignoring the challenger.

Related Literature

Most existing models of accountability fall into three categories: (i) theories in which all politicians are identical and the challenger is simply a replacement for the incumbent, (ii) theories in which politicians are heterogeneous and the challenger is a replacement drawn from the same pool as the incumbent, and (iii) theories in which politicians are heterogeneous and the challenger is drawn from a different pool than the incumbent. Note that in all three canonical modeling setups the “challenger” is a completely passive replacement, rather than an active player.

In a few models, challengers do take actions, typically by deciding whether to enter the race in the first place. Entry has been analyzed as something that incumbents seek to deter (Epstein and Zemsky 1995, Goodliffe 2005), or as a costly signal that the challenger knows herself to be high quality (Gordon, Huber, and Landa 2007). In a different vein, Kramer (1977) models challengers as adopting platforms in a sequence of elections between two parties. Although these models provide valuable insights into some aspects of electoral competition, challengers in these models only take the most minimal types of actions—entering the race or declaring platforms. Unlike the challenger in our model, they do not actively assess or criticize the incumbent’s performance.

The first model we are aware of in which challengers make statements about what policies best serve voters’ interests is Lemon (2005). In that model, the challenger makes an

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1 The literature is far too vast to review fully. The first category includes Ferejohn (1986) and Gordon and Huber (2002). The second includes Rogoff (1990), Maskin and Tirole (2004), Fox (2007), Banks and Duggan (2008), and Acemoglu, Egorov and Sonin (2013). The third includes Banks and Sundaram (1993), Canes-Wrone, Herron, and Shotts (2001), and Ashworth and Bueno de Mesquita (2008).
announcement either previous to or simultaneous with the incumbent’s policy choice. This sequence of moves is contrary to the ordinary sequence of politics—in which incumbents take action and then opponents offer criticisms—and thus cannot be used to model challengers as attacking the incumbent’s policy record.

A model in which challengers move after incumbents is developed in a recent paper by Dewan and Hortala-Vallve (2014), who analyze how voters learn about the competence of incumbents (via their policies) and challengers (via their campaign platforms). A key difference is that in their model a challenger’s campaign reveals information about his own competence and don’t reveal anything about the incumbent or her policies, whereas in our model the role of the challenger is to criticize the incumbent and her policy choices.

The theory most closely related to ours is Warren’s (2012) model of an auditor, e.g., the media, who can obtain a verifiable signal about the incumbent’s private information. Warren’s model focuses on control of incumbents who have non-congruent policy preferences, whereas our focus is on the incumbent’s effort to learn what policies best promote voters’ interests. More importantly, his model focuses on auditors who have independent incentives to acquire and reveal information about the incumbent’s policymaking, rather than caring about the outcome of the election. In his model, if the auditor’s sole goal is to cause the incumbent to lose the election (which is an electoral challenger’s goal), then the auditor’s presence cannot improve the incumbent’s incentives to choose policies that promote voters’ interests. In our model, in contrast, monitoring by a challenger can result in improved incumbent policymaking.

The Model

The model consists of two governance periods, with an election in between. There are three actors: two office-motivated politicians (incumbent and challenger) and a voter who tries to elect politicians who will make good policy choices. In the first governance period, the incumbent is in office. Then the voter decides whether to retain the incumbent or elect the challenger.

In each governance period, the politician in power chooses a policy $x^t \in \{A, B\}$. The voter wants the policy to match the state of the world $\omega^t \in \{A, B\}$, receiving utility 1 if $x^t = \omega^t$ and 0 otherwise. The states are independent across time, and each state is equally likely. Initially, no one knows the state for either period. In each governance period, each politician observes two signals about the state. The accuracy of these signals depends on the politician’s type $\theta_i \in \{G, N\}$. The incumbent is Good with probability
Pr(θ_I = G) = β, where 0 < β < 1 and Normal with probability 1 − β. Similarly, for the challenger, Pr(θ_C = G) = γ, where 0 < γ < 1. At the beginning of the game, neither the politicians nor the voters know any politician’s type.

One signal that a politician receives, y_t^i ∈ {A, B}, is an impression that is free and non-verifiable. The probability that an impression is accurate depends on the politician’s type. A good type’s impression is always correct: Pr(y_t^i = ω_t^i | θ_i = G) = 1. A normal type’s impression is a coin flip: Pr(y_t^i = ω_t^i | θ_i = N) = 1/2.²

In addition, each politician can invest effort in investigation. Specifically, politician i in period t gets signal s_t^i ∈ {A, B, φ}. This signal is either perfect, i.e., s_t^i = ω_t, or completely uninformative, s_t^i = φ independent of the state. Investigative signals s_t^i ∈ {A, B} are verifiable by the voters, and cannot be faked. Politicians cannot prove, however, that they got no information. This means that a candidate who doesn’t have any hard information available must report φ.

The probability of an informative investigative signal depends on the politician’s quality, whether she holds office, and her effort. We assume that more effort leads to a higher probability of receiving an informative investigative signal, and this probability is also weakly higher if the politician is a good type. The incumbent pays an effort cost c(q) and the probability that she learns the true state is q if she is good and α_Iq if she is normal (where α_I ≤ 1). The challenger pays a cost k(r) to learn the true state with probability r.³ Both c(·) and k(·) are twice continuously differentiable with strictly positive first and second derivatives (except possibly at 0), satisfy c(0) = k(0) = 0, and the Inada conditions: c'(0) = k'(0) = 0, c'(1) > 1, and k'(1) > 1. Politicians are also office-motivated, receiving utility 1 for each period in office and 0 when out of office.

The voter’s only source of information after the first governance period is the politicians’ announcements. The sequence of the model is as follows.

**Date 1 Governance period 1**

1.1 Incumbent and challenger simultaneously choose q and r

1.2 Incumbent and challenger observe signals y_I ∈ {A, B}, s_I ∈ {A, B, φ}, y_C ∈ {A, B}, and s_C ∈ {A, B, φ}

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²These assumptions about the accuracy of the impression can be generalized substantially.
³A more general model would have success probabilities r and α_CR for a good and normal challenger, with α_C ∈ [0, 1]. Because our focus is on voter learning about the incumbent, not the challenger, we set α_C = 1. At a technical level, this assumption simplifies belief updating and eliminates the possibility of some implausible equilibria (e.g., in which the challenger wins the election by proving that the incumbent’s policy choice was correct).
1.3 Incumbent chooses policy $x^1 \in \{A, B\}$

**Date 2** Campaign and election

2.1 Incumbent and challenger make simultaneous announcements $m_I \in \{s_I, \phi\}$ and $m_C \in \{s_C, \phi\}$

2.2 Voter observes policy and announcements and chooses the election winner

**Date 3** Governance period 2

3.1 Election winner chooses $q$, and observes signals

3.2 Election winner chooses policy

Two things about the sequence are worth noting. Although we model simultaneous incumbent and challenger announcements at Date 2, the announcements could be sequential. For example, the equilibrium is unchanged if the incumbent makes an announcement when choosing policy at stage 1.3, and then the challenger makes an announcement during the campaign at stage 2.1. Also, note that we don’t allow the out-of-power politician to make announcements in the second governance period. This assumption makes no difference, because the out-of-power politician would have no incentive to exert effort, and it saves on notation as we analyze the first governance period and the election.

**Equilibrium**

Pure strategies in our model are as follows.

**Incumbent**

- Effort: $q \in [0, 1]$
- Policy: $x^1(q, y_I, s_I): [0, 1] \times \{A, B\} \times \{A, B, \phi\} \rightarrow \{A, B\}$
- Announcement: $m_I(q, y_I, s_I, x^1): [0, 1] \times \{A, B\} \times \{A, B, \phi\} \times \{A, B\} \rightarrow \{A, B, \phi\}$
  with $m_I(q, y_I, s_I, x^1) \in \{s_I, \phi\}$

**Challenger**

- Effort: $r \in [0, 1]$
- Announcement: $m_C(r, y_C, s_C, x^1): [0, 1] \times \{A, B\} \times \{A, B, \phi\} \times \{A, B\} \rightarrow \{A, B, \phi\}$
  with $m_C(r, y_C, s_C, x^1) \in \{s_C, \phi\}$
Voter

- Election decision \( p(x^1, m_I, m_C) : \{A, B\} \times \{A, B, \phi\}^2 \rightarrow \{I, C\} \)

- Belief about incumbent type: \( \mu(x^1, m_I, m_C) : \{A, B\} \times \{A, B, \phi\}^2 \rightarrow [0, 1] \)

**Equilibrium concept** We focus on perfect Bayesian equilibria that satisfy three additional requirements. First, the two policies are treated symmetrically. Second, the incumbent always chooses the policy that is best for the voter given all of her information. This implies that an incumbent who conducts a successful investigation chooses \( x^1 = s_I \) and an incumbent whose investigation produces no information chooses \( x^1 = y_I \). Third, the voter’s belief that the incumbent is a good type, \( (x^1; m_I; m_C) \), satisfies respect for evidence:

- \( \mu(x^1, m_I, m_C) = 0 \) if \( m_C \neq \phi \) and \( m_C \neq x^1 \)
- \( \mu(x^1, m_I, \phi) = \frac{\beta}{\beta + (1-\beta)\alpha_I} \) for all \( x^1 \) and \( m_I \in \{A, B\} \).

We say that an equilibrium is **competitive** if each candidate has a strictly positive probability of winning, and **non-competitive** otherwise.

**Passive Replacement**

We begin by analyzing a simplified version of our model, in which the challenger cannot make investigations or announcements, but rather simply acts as a replacement. This is what is assumed in standard accountability models, and it serves as a baseline for analyzing the effect of a challenger who can actively criticize the incumbent. The model with a passive replacement has the following structure.

**Date 1** Governance period 1

1.1 Incumbent chooses \( q \) and observes signals \( y_I \in \{A, B\} \) and \( s_I \in \{A, B, \phi\} \)

1.2 Incumbent chooses policy \( x^1 \in \{A, B\} \)

**Date 2** Campaign and election

2.1 Incumbent makes announcement \( m_I \in \{s_I, \phi\} \)

2.2 Voter observes the policy and announcement and chooses the election winner
3.1 Election winner chooses \( q \), and observes signals

3.2 Election winner chooses policy

The model with a passive replacement features two types of equilibria: a competitive direct accountability equilibrium in which the election outcome depends on the incumbent’s announcement, as well as noncompetitive equilibria in which the election outcome is determined by the voter’s prior beliefs about the two politicians.

In a direct accountability equilibrium, the incumbent is re-elected if and only if she announces information proving that her policy choice was correct.\(^4\) The incumbent announces \( m_I = x^1 \) whenever her information yields hard information, and she sets an effort level \( q_0^* \) that balances marginal costs of effort against the marginal probability of obtaining hard information, i.e., such that

\[
c'(q_0^*) = \beta + (1 - \beta) \alpha_I.
\]  

Two conditions must hold for a direct accountability equilibrium to exist: the voter must prefer to re-elect the incumbent when \( m_I = x^1 \) and he must prefer to remove her when \( m_I = \phi \).

For the voter to re-elect the incumbent when \( m_I = x^1 \) requires that his prior belief about the replacement be lower than his updated belief about the incumbent:

\[
\gamma \leq \mu(x^1, m_I = x^1) = \frac{\beta q_0^*}{\beta q_0^* + (1 - \beta) \alpha_I q_0^*} \equiv \bar{\mu}_0
\]  

(2)

Note that for any effort level \( q_0^* \in (0, 1) \), \( \bar{\mu}_0 \geq \beta \), with a strict inequality for \( \alpha_I < 1 \).

For the voter to remove the incumbent when \( m_I = \phi \) requires that his prior belief about the replacement be greater than his updated belief about an incumbent who doesn’t prove that her policy choice is correct:

\[
\gamma \geq \mu(x^1, m_I = \phi) = \frac{\beta (1 - q_0^*)}{\beta (1 - q_0^*) + (1 - \beta) (1 - \alpha_I q_0^*)} \equiv \mu_0
\]  

(3)

Note that for any effort level \( q_0^* \in (0, 1) \), \( \mu_0 \leq \beta \), with a strict inequality for \( \alpha_I < 1 \).

\(^4\)When there’s a pure strategy direct accountability equilibrium, there’s also a continuum of similar equilibria involving mixing. Specifically, if the incumbent announces \( m_I = x^1 \) she’s re-elected for sure. If she makes no announcement, she wins with probability \( \sigma \in (0, 1) \) such that her optimal \( q \in (0, q_0^*) \) induces voter beliefs \( \mu(x^1, m_I = \phi|q) = \gamma \).
Combining Equations 2 and 3, we see that a direct accountability equilibrium exists whenever the passive replacement is ex ante close to the incumbent, i.e., $\gamma \in \left[\mu_0, \mu_0 \right]$. If the incumbent produces hard information, the voter updates positively about her quality and re-elects her. If the incumbent fails to produce hard information the voter updates down and removes her.

In a noncompetitive equilibrium, the outcome simply depends on the voter’s prior. In such an equilibrium, the incumbent exerts zero effort on policymaking. If the replacement is behind the incumbent ex ante, i.e., $\gamma < \beta$, there exists a noncompetitive equilibrium, in which the incumbent always wins. Note that this means that for $\gamma \in \left[\mu_0, \beta \right]$, there are two equilibria, one of which is competitive and one of which is noncompetitive. Clearly the competitive one is better for the voter, both in terms of first period policymaking and in terms of selection.

If the replacement is ahead of the incumbent, i.e., $\gamma > \beta$, existence of a noncompetitive equilibrium depends on how far behind the incumbent is, because she can induce the voter to update positively by announcing hard information. Specifically, from Equation 2 we see that if $\gamma < \bar{\mu}_0$ then an incumbent who produces hard information wins re-election. Because $c'(0) = 0$ there cannot exist a noncompetitive equilibrium in which the incumbent always loses. However, if the replacement is farther ahead of the incumbent ($\gamma > \bar{\mu}_0$) then there exists a noncompetitive equilibrium in which she always loses.

In summary, we have the following pattern of equilibria, as shown in Figure 1a.

**Proposition 1** With a passive replacement, there exist the following types of equilibria, depending on the voter’s prior belief about the replacement.

1. For $\gamma \leq \beta$, there is a noncompetitive equilibrium. The incumbent exerts no effort and always wins the election.
2. For $\gamma \in \left[\mu_0, \bar{\mu}_0 \right]$, there is a direct accountability equilibrium. The incumbent exerts effort $q^*_0$ and wins re-election iff $m_1 = x^1$.
3. For $\gamma \geq \bar{\mu}_0$, there is a noncompetitive equilibrium. The incumbent exerts no effort and always loses the election.

**Active Challenger**

We now turn to our main model, in which the challenger receives a soft-information impression and hard information based on his investigation. Note that the challenger’s impression
cannot play any role in the election, because if it did then the challenger would have an incentive to misreport it in whatever way would maximize his probability of winning. We begin by determining whether the equilibria characterized in Proposition 1 continue to exist with an active challenger rather than a passive replacement.

1. The noncompetitive equilibrium in which the incumbent always wins is no longer an equilibrium. The reason for this is that the challenger can win the election by obtaining hard information proving that the incumbent’s policy choice was incorrect (which means that the incumbent must be normal). Because a normal incumbent chooses the wrong policy with strictly positive probability and \( k'(0) = 0 \), the challenger is willing to exert a strictly positive level of effort, \( r > 0 \), thereby breaking the equilibrium.

2. The direct accountability equilibrium is still an equilibrium. To see why, note that when the incumbent does not announce hard information (\( m_I = \phi \)) the challenger wins the election. And when the incumbent announces \( m_I = x^1 \), the challenger can’t induce the voter to update negatively on the incumbent because the incumbent’s policy choice is correct. Thus the challenger has no reason to exert any effort, and sets \( r = 0 \). The parameter values for which this equilibrium exists are the same as in the passive replacement model: \( \gamma \in [\bar{\mu}_0, \bar{\mu}_0] \).

3. The noncompetitive equilibrium in which the incumbent always loses is still an equilibrium, with the challenger setting \( r = 0 \). The parameter values for which this equilibrium exists are the same as in the passive replacement model (\( \gamma \geq \bar{\mu}_0 \)).

The main implication of these three points is that an active challenger who is behind the incumbent poses a threat to her. If the challenger is just a bit behind the incumbent ex ante, i.e., \( \gamma \in [\bar{\mu}_0, \beta] \), the presence of an active challenger eliminates one possible equilibrium (the noncompetitive one in which the incumbent doesn’t work and is always re-elected). If the challenger is substantially behind the incumbent ex ante, i.e., \( \gamma < \bar{\mu}_0 \), the noncompetitive equilibrium is the only equilibrium in the passive-replacement model, and it ceases to be an equilibrium if the challenger is an active player.

There is also another type of equilibrium that can occur with an active challenger: a monitoring equilibrium, in which the incumbent is re-elected unless the challenger proves that she chose the wrong policy.\(^5\)

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\(^5\)This is the only other possible type of pure strategy equilibrium. If the challenger proves the incumbent’s policy choice to be incorrect, then the incumbent is normal and must lose the election. Thus, there are only two types of information sets (\( m_I = x^1 \) and \( m_I = m_C = \phi \)) where the voter can either reelect or remove the
In a monitoring equilibrium, investigative efforts are determined as a Nash equilibrium of a simultaneous effort choice game played by the incumbent and challenger, with payoffs calculated as follows. An incumbent who is good always chooses the correct policy. The incumbent is normal with probability \((1 - \beta)\), in which case she fails to obtain hard information with probability \(1 - q_{\alpha I}\), and chooses the incorrect policy with probability \(\frac{1}{2}\). The challenger obtains hard information with probability \(r\). Thus the probability that the incumbent loses is \(r (1 - q_{\alpha I}) \frac{1 - \beta}{2}\), and efforts in a monitoring equilibrium are a Nash equilibrium \((q^*, r^*)\) of the simultaneous move game with incumbent payoff

\[
1 - r (1 - q_{\alpha I}) \frac{1 - \beta}{2} - c(q)
\]

and challenger payoff

\[
r (1 - q_{\alpha I}) \frac{1 - \beta}{2} - k(r).
\]

Existence of a monitoring equilibrium follows from standard arguments and uniqueness can be established using the index theorem (Vives 1999, p. 48). The Inada conditions imply that the equilibrium is interior, so the first-order conditions

\[
q^* \alpha I \frac{1 - \beta}{2} = c^*
\]

\[
(1 - q^* \alpha I) \left( \frac{1 - \beta}{2} \right) = k^*
\]

fully characterize the politicians’ efforts.

To determine when a monitoring equilibrium exists, we need to consider two possibilities for the incumbent’s actions when her investigation yields hard information. One possibility is that the incumbent chooses \(x^1 = s_I\) but doesn’t publicly reveal her hard information, and simply reports \(m_I = \phi\). The other possibility is that the incumbent reports \(m_I = x^1 = s_I\). For sufficiently low voter beliefs about the challenger, either of these strategies can be part of a monitoring equilibrium, and for sufficiently high voter beliefs neither strategy can be part of a monitoring equilibrium. The exact cutpoint for each strategy depends on other parameters of the model, as follows.

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\(\alpha\) is the probability of the incumbent being normal. Reelect-remove is direct accountability, reelect-reelect and remove-reelect are monitoring, and remove-remove is noncompetitive with the challenger always winning.

\(6\)It is also possible to have monitoring equilibria in which the incumbent reveals hard information with some probability \(\gamma \in (0, 1)\). The set of \(\gamma\) values for which such an equilibrium exists is the same as what we characterize here.
Case 1. Suppose the incumbent reveals hard information, i.e., \( m_I = s_I \) whenever \( s_I \neq \phi \). For what \( \gamma \)'s does such a monitoring equilibrium exist? Two conditions must be satisfied.

The first condition is that the voter’s prior belief about the challenger must be sufficiently low so that after the incumbent reveals hard information, she is reelected, i.e., as in Equation 2 from our analysis of the passive replacement model, \( \gamma \leq \bar{\mu}_0 \).

The second condition is that \( \mu \left( x^1, \phi, \phi \right) \geq \gamma \), i.e., the voter’s belief about the incumbent when neither politician reveals hard information must be higher than his prior about the challenger. To calculate this belief, note that a high-quality incumbent fails to find hard information with probability \( 1 - q^* \), and because her policy choices are always correct the challenger never obtains information indicating they’re wrong. A normal incumbent fails to find hard information with probability \( 1 - q^* \alpha_I \). When she fails to find hard information, her policy choices are incorrect with probability \( 1/2 \), and the challenger’s probability of obtaining \( s_C \neq x^1 \) is \( r^* \). Thus the voter’s belief is

\[
\mu \left( x^1, \phi, \phi \right) = \frac{\beta \left( 1 - q^* \right)}{\beta \left( 1 - q^* \right) + \left( 1 - \beta \right) \left( 1 - q^* \alpha_I \right) \left( 1 - 1/2 r^* \right)} = \bar{\mu}_{reveal}.
\tag{5}
\]

Combining the two conditions, we see that to have a monitoring equilibrium with \( m_I = s_I \) when \( s_I \neq \phi \) requires that \( \gamma \leq \min \{ \bar{\mu}_0, \bar{\mu}_{reveal} \} \).

Case 2. Suppose the incumbent doesn’t reveal hard information, i.e., \( m_I = \phi \) even when \( s_I \neq \phi \). For what \( \gamma \)'s does such a monitoring equilibrium exist? This requires \( \gamma \leq \mu \left( x^1, \phi, \phi \right) \), i.e., the voter’s belief about the incumbent when neither politician reveals hard information must be higher than his prior about the challenger. Because the incumbent does not report hard information, this belief is calculated differently from Case 1, as follows. A good incumbent never reveals any information, and because her policy choices are always correct the challenger never obtains contradictory information. A normal incumbent never reveals any information. With probability \( \frac{1 - q^* \alpha_I}{2} \) her policy choices are wrong and the challenger’s probability of obtaining \( s_C \neq x^1 \) is \( r^* \). Thus the voter’s belief is

\[
\mu \left( x^1, \phi, \phi \right) = \frac{\beta}{\beta + \left( 1 - \beta \right) \left( 1 - \frac{1 - q^* \alpha_I}{2} r^* \right)} = \bar{\mu}_{noreveal}
\tag{6}
\]

Note that \( \bar{\mu}_{noreveal} > \beta \) because \( q^* \alpha_I < 1 \) and \( r^* > 0 \). To have a monitoring equilibrium in which the incumbent always reports \( m_I = \phi \) requires \( \gamma \leq \bar{\mu}_{noreveal} \).

From Case 1 and Case 2 we see that a monitoring equilibrium exists if and only if the voter’s belief about the challenger is sufficiently low: \( \gamma \leq \max \{ \min \{ \bar{\mu}_0, \bar{\mu}_{reveal} \}, \bar{\mu}_{noreveal} \} \).
Note, however, that “sufficiently low” includes some values of $\gamma$ such that the challenger is ex ante ahead of the incumbent, because $\bar{\mu}_{\text{noreveal}} > \beta$.

The relative values of the three cutoffs depend on the other parameters of the model. However, a necessary and sufficient condition for existence of a monitoring equilibrium is $\gamma \leq \bar{\mu}_{\text{noreveal}}$, because $\bar{\mu}_{\text{noreveal}} \leq \bar{\mu}_0$ iff $\bar{\mu}_{\text{reveal}} \leq \bar{\mu}_{\text{noreveal}}$. To prove this claim, we first combine Equations 2 and 6 as follows:

$$
\frac{\beta}{\beta + (1 - \beta)\left(1 - \frac{1 - q^*\alpha_I r^*}{2}\right)} \leq \frac{\beta q_0^*}{\beta q_0^* + (1 - \beta) \alpha_I q_0^*},
$$

$$
\alpha_I \left(1 - \frac{q^* r^*}{2}\right) \leq 1 - \frac{r^*}{2}.
$$

(7)

Next, we combine Equations 5 and 6 to obtain the same condition:

$$
\frac{\beta (1 - q^*)}{\beta (1 - q^*) + (1 - \beta) (1 - q^* \alpha_I) \left(1 - \frac{1 - q^* r^*}{2}\right)} \leq \frac{\beta}{\beta + (1 - \beta) \left(1 - \frac{1 - q^* \alpha_I r^*}{2}\right)},
$$

$$
\alpha_I \left(1 - \frac{q^* r^*}{2}\right) \leq 1 - \frac{r^*}{2}.
$$

Henceforth we denote the cutoff for existence of a monitoring equilibrium as $\bar{\mu} \equiv \bar{\mu}_{\text{noreveal}}$. Note that it’s possible to have either $\bar{\mu} > \bar{\mu}_0$ or $\bar{\mu} < \bar{\mu}_0$, depending on the parameters of the model. In particular, from Equation 7 we see that $\bar{\mu} > \bar{\mu}_0$ if and only if

$$
\alpha_I > \frac{2 - r^*}{2 - q^* r^*}.
$$

(8)

Intuitively, when the incumbent’s ability has little effect on the probability that she obtains hard information ($\alpha_I \approx 1$), the voter doesn’t update upwards very much when the incumbent produces hard information. Thus if the challenger is only a bit ahead of the incumbent ex ante, there can be a monitoring equilibrium but not a direct accountability one. However, if $\alpha_I$ is low, the voter updates upwards a lot when the incumbent produces hard information, and thus when the challenger is substantially ahead of the incumbent it’s possible to have a direct accountability equilibrium, but not a monitoring one.

In summary, we have the following pattern of equilibria with a challenger who can gather information and criticize the incumbent’s policy choices, as shown in Figure 1b.
Proposition 2  With an active challenger, there exist the following types of equilibria.

1. For $\gamma \leq \bar{\mu}$, there is a monitoring equilibrium. The incumbent sets $q = q^*$ and the challenger sets $r = r^*$. The incumbent wins re-election unless the challenger proves her policy choice was incorrect, i.e., $m_C \notin \{x^1, \phi\}$.

2. For $\gamma \in [\hat{\mu}_0, \bar{\mu}_0]$ there is a direct accountability equilibrium. The incumbent sets $q = q_0^*$ and the challenger sets $r = 0$. The incumbent wins re-election iff $m_I = x^1$.

3. For $\gamma > \bar{\mu}_0$ there is a noncompetitive equilibrium. The incumbent exerts no effort and always loses the election.

Effect of Challenger

We now compare the equilibria in Propositions 1 and 2 to determine how the presence of an active challenger affects the incumbent’s policymaking incentives as well as selection of the politician for the second governance period.

As a first step for this analysis, we compare the monitoring equilibrium with the equilibria that arise with a passive replacement.

It is obvious that monitoring is better than a noncompetitive equilibrium, for both incentives and selection.

Compared to a direct accountability equilibrium, monitoring is worse for incentives ($q^* < q_0^*$, which is straightforward to see from Equations 1 and 4). This is intuitive: with direct accountability an incumbent has a substantial electoral incentive to produce hard information whereas with monitoring hard information is only useful to an incumbent in a limited set of circumstances (specifically that if she didn’t have hard information she would have chosen the wrong policy and been caught by the challenger).

For selection, monitoring can be either better or worse than direct accountability. To see this we consider the special case $\beta = \gamma$ and $\alpha_I < 1$, in which case both types of equilibria exist. In a direct accountability equilibrium, the probability that the second period politician is high quality is

$$\beta q_0^* + \gamma [1 - q_0^* (\beta + (1 - \beta) \alpha_I)] = \beta q_0^* + \beta [1 - q_0^* (\beta + (1 - \beta) \alpha_I)]$$

In a monitoring equilibrium, the probability that the second period politician is high quality is

$$\beta + \gamma (1 - \beta) \left( \frac{1 - \alpha_I q^*}{2} \right) r^* = \beta + \beta (1 - \beta) \left( \frac{1 - \alpha_I q^*}{2} \right) r^*$$
To see that monitoring can be better for selection, consider the following setup. Let the cost function for the incumbent be \( C \cdot c(q) \). For \( C > 1 \) the Inada conditions are satisfied, and for “large” values of \( C \), \( q_0^* \approx 0 \) and \( q^* \approx 0 \). Also note that \( r^* \) does not go to zero, because the challenger’s probability of catching an incumbent mistake is highest when the incumbent puts little effort into policymaking. Thus, the probability that the second period politician is good in the direct accountability equilibrium is simply \( \gamma = \beta \) whereas in the monitoring equilibrium it is strictly higher: \( \beta + \gamma (1 - \beta) \left( \frac{1}{2} \right) r^* \). The intuition behind this example is that with no incumbent effort, direct accountability is useless for selection whereas monitoring enables the voter to select based on information that the challenger gathers about the incumbent’s impression-based policy choices.

To see that direct accountability can be better for selection, return to the setting with \( C = 1 \) and let \( \alpha_I \) be very small. Then from Equation 1, \( q_0^* \) is bounded away from 0. Thus the probabilities of having good second period politicians are \( \beta + \beta (1 - \beta) q_0^* \) and \( \beta + \beta (1 - \beta) \left( \frac{r^*}{2} \right) \) for direct accountability and monitoring, respectively, and as long as the parameters are such that \( q_0^* > \frac{r^*}{2} \) (which can easily be done, e.g., with quadratic costs) then direct accountability is superior for selection.

Having compared the different types of equilibria in our model, we now compare parameter regions from Propositions 1 and 2, to determine the effects of having an active challenger rather than a passive replacement.

- If the challenger is behind the incumbent ex ante (\( \gamma < \beta \)) then having an active challenger eliminates the non-competitive passive-replacement equilibrium and thus can improve both incentives and selection.
  
  - If the challenger is sufficiently far behind, \( \left( \gamma < \mu_0 \right) \), then with a passive replacement the unique equilibrium is noncompetitive whereas with an active challenger the unique equilibrium is monitoring, so both incentives and selection are better with the challenger.

- If the challenger is close to the incumbent ex ante \( \left( \gamma \in [\mu_0, \mu_0] \right) \) then having a challenger may make no difference, as long as the voters ignore him and make decisions directly based on the incumbent’s behavior and announcements. However, it also is possible that the voters use challenger announcements to monitor the incumbent; this makes incumbent incentives worse and may either improve or worsen selection.

- If an incumbent’s acquisition of hard information is sufficiently uninformative about her type \( \left( \alpha_I > \frac{2 - r^*}{2 - q^* r^*} \right. \) from Equation 8), and the challenger is moderately far ahead
of the incumbent \((\gamma \in (\bar{\mu}_0 < \bar{\mu}))\), then having a challenger who actively monitors the incumbent improves both incentives and selection, relative to the noncompetitive equilibrium in the passive-replacement model.

- If the challenger is sufficiently far ahead, \((\gamma > \max\{\bar{\mu}, \bar{\mu}_0\})\), then having a challenger makes no difference for either incentives or selection because the only equilibrium is a noncompetitive one.

Overall, our results are only partially consistent with the intuition, outlined in the introduction, that monitoring by challengers is good for electoral accountability. Monitoring clearly is better than having policymakers who are unaccountable and unreplaceable. But when the challenger and incumbent are ex ante similar, monitoring can be worse than holding the incumbent directly accountable for her actions. And the reason it can be worse is that with monitoring the incumbent can actually have too much of an incumbency advantage, in the sense that she is re-elected unless her policy choices are proven to be wrong.

In contrast, when the incumbent and challenger are unevenly matched, direct accountability is infeasible and monitoring is beneficial. In these circumstances, having an active challenger makes it possible to have at least some accountability, thereby improving both incumbent incentives and selection.

**Information in Open Seat Elections**

*<This section is particularly rough & preliminary>*

We now extend our model by adding an initial stage, in which the voter decides between two candidates in an open-seat election. Each candidate is a good type with prior probability \(\gamma\). Prior to voting in the open seat election, the voter receives an informative signal about each open seat candidate. The posterior following the signal is \(\beta \in \{\bar{\beta}, \beta\}\), where \(\bar{\beta} - \gamma = \gamma - \bar{\beta} \equiv \delta > 0\). (Consistency with Bayesian updating then requires each posterior is equally likely ex-ante.) The model is unchanged aside from the following addition at the beginning of the game sequence:

**Date 0** The open-seat election occurs:

0.1 The voter observes a signal about each open seat candidate.
0.2 The voter chooses the winner of the election.
In the previous section we characterized equilibria for subgames starting at Date 1. We impose one additional requirement on the model: that the open-seat election candidates are treated symmetrically. This implies both that they have equal chances of winning if their signals are both good or both bad, and that equilibrium selection in the continuation games cannot depend on the identity of the open-seat winner.

If the voter always chooses the open-seat candidate that is more likely to be high quality (something that needs to be verified as being consistent with equilibrium), then \( \frac{1}{4} \) of the time the voter’s posterior on the open-seat election winner is \( \beta \) and the other \( \frac{3}{4} \) of the time it is \( \bar{\beta} \). Much of the notation in previous sections now needs to be parametrized in terms of the voter’s belief after date 0 about the open-seat election winner \( ( \beta \text{ or } \bar{\beta} ) \).

We use the parameter \( \delta \) to characterize the informativeness of the open-seat election, and we focus on two types of open-seat elections: essentially-uninformative (\( \delta \) close to 0) and highly-informative (\( \delta \) “sufficiently large”).

**Low-Information Open Seat Elections**

For \( \delta \) sufficiently close to 0, \( \mu_0 ( \beta ) < \gamma < \tilde{\mu}_0 ( \beta ) \) and \( \mu_0 ( \bar{\beta} ) < \gamma < \tilde{\mu}_0 ( \bar{\beta} ) \), i.e., regardless of whether the voter updated up or down about the incumbent after Date 0, there exists a direct accountability equilibrium in the continuation game. Likewise, for \( \delta \) sufficiently close to 0, \( \gamma < \tilde{\mu} ( \beta ) \) and \( \gamma < \tilde{\mu} ( \bar{\beta} ) \), i.e., regardless of whether the voter updated up or down about the incumbent after Date 0, there exists a monitoring equilibrium in the continuation game.

These equilibria feature very different properties. With direct accountability, the incumbent works relatively hard, the challenger is ignored, and the probability that the incumbent is re-elected can be either high or low (depending on the equilibrium \( q_0^* \)). With monitoring, the incumbent doesn’t work as hard, the challenger plays a crucial role in the voter’s election decisions, and it is straightforward to see that the probability that the incumbent is re-elected is strictly greater than 1/2.

This observation has implications for interpretation of regression-discontinuity studies of the incumbency advantage. If \( \delta \approx 0 \), all open seat races in our model are “close”. So the model predicts different patterns of incumbency advantage depending on which equilibrium is being played.

With a direct accountability equilibrium, if the incumbent’s cost function \( c ( \cdot ) \) is sufficiently steep close to \( q = 0 \) then incumbents exert little effort, and there is a negative incumbency advantage, as has been observed empirically in some countries (e.g., India). On the other hand, if costs are low, the incumbent is likely to produce solid information in
support of her policy choice, there is a positive incumbency advantage.

With a monitoring equilibrium, the incumbency advantage is always positive (i.e., the incumbent is always re-elected with probability strictly greater than \( \frac{1}{2} \)).

**High-Information Open Seat Elections**

For \( \delta \) sufficiently high, the model predicts a unique equilibrium. If the voter receives bad signals about both open-seat candidates, then the voter’s belief about the incumbent is \( \beta \). For sufficiently high \( \delta \), \( \hat{\mu} (\beta) < \gamma \), i.e., the only equilibrium of the continuation game is a noncompetitive equilibrium, with the incumbent exerting zero effort and the challenger always winning. This represents a substantial incumbency disadvantage. Also, for sufficiently high \( \delta \), \( \gamma < \hat{\mu}_0 (\beta) \), i.e., the only equilibrium of the continuation game is a monitoring equilibrium, with the incumbent exerting strictly positive effort and winning re-election with probability strictly greater than \( \frac{1}{2} \).

**Soft Information**

To see why hard information is important in our model, we now consider a variant with only soft information. Specifically, we now suppose that politicians’ investigations yield information–\( s_I \in \{A, B, \phi\} \) and \( s_C \in \{A, B, \phi\} \)–that is soft, rather than hard. As in our main model, the politicians’ impressions are soft information and the voter’s only information when making election decisions is the policy choice \( x^1 \) and the politicians’ announcements \( m_I \) and \( m_C \), which are unconstrained (i.e., \( m_I \in \{A, B, \phi\} \) and \( m_I \in \{A, B, \phi\} \) regardless of the politicians’ information).

At Date 2, given the incumbent’s policy choice \( x^1 \), the incumbent and challenger are playing a simultaneous move zero-sum game, with payoffs that are the vector of probabilities \( p_{x^1 m_I m_C} \) that the incumbent wins, as a function of her policy choice \( x^1 \) and the politicians’ announcements \( m_I \) and \( m_C \). The equilibrium to this game depends solely on \( x^1 \) and the election probabilities, and does not depend on the state of the world or the politicians’ information \( y_I, s_I, y_C, \) and \( s_C \). Let \( p (x^1) \) denote the incumbent’s probability of winning in this game, given \( x^1 \), and note that at Date 1, the incumbent obviously will choose the policy \( x^1 \in \{A, B\} \) with the maximal \( p (x^1) \). Thus, there is no reason for either actor to invest in effort, and they both set \( q = r = 0 \). With these efforts, the equilibrium that yields the maximal voter payoff is one in which the incumbent chooses policy based on her impression, \( x^1 = y_I \), and the voter makes his electoral decision based on his priors, \( \beta \) and \( \gamma \). This same
pattern of policymaking and electoral behavior is also possible if the challenger is a passive replacement; thus, having an active challenger does not make the voter better off.\footnote{We plan to try to generalize this argument by altering the model in this section by having the voter see a noisy public signal after the politicians’ announcements and before the election. With a public signal, the voter can induce accurate challenger information-revelation by making decisions based on his announcement, but that doesn’t necessarily imply that he can simultaneously improve incumbent incentives or selection.}

### Conclusion

*<To be written once analysis of model is complete.>*

### References


Figure 1a: Equilibria With Passive Replacement

Figure 1b: Equilibria With Active Challenger

Equilibrium regions are shown as a function of prior on challenger, $\gamma$, ranging from 0 to 1.

With active challenger, equilibria are shown for the case where $\alpha$ is sufficiently high so that $\mu > \mu_0$. 