Delay in Contests

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ABSTRACT

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Why is there delay in contests? In this paper we follow and extend the line of reasoning of Carl von Clausewitz to explain delay. For a given contest technology, delay may occur if there is an asymmetry between defense and attack, if the expected change in relative strengths is moderate, and if the additional cost of investment in future strength is low.

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1 Introduction

In conflict situations the showdown is often delayed, even if it seems clear that the showdown cannot be avoided. In movies and in writings delay of the showdown is a common pattern. This reflects similar patterns of conflict in the real world, in the area of military conflict and other conflictual situations.

Suppose there are two contestants, A and B. The contestants know their current relative strengths and anticipate their expected future strengths, where strength can be seen as a broad measure of conflict resources, intelligence about the rival's resources etc. If both agents are fully informed about the status quo and all future changes of the status quo are deterministic and fully known to the agents, the situation cannot improve for both of them. This is what generates a puzzle: As Carl von Clausewitz (1832/1976, p.84) puts it: "If it is in A's interest not to attack B now but to attack him in four weeks, then it is in B's interest not to be attacked in four weeks' time, but now."

Clausewitz also offers a solution. He needs two basic insights for his conclusions: first, each contestant can force the showdown, but only as an attacker. No doubt, the showdown between A and B takes place if one party, say A, starts it, and B has no reasonable[1] option other than to fight. Second, there is an asymmetry between attack and defense, and it is advantageous to take the role of defense. Based on his knowledge of military history and on his personal experience as a military leader Clausewitz argues that the claim often made that the attacker has the advantage is wrong in most situations.[2]

In line with Clausewitz' insights is a more recent example for conventional war. Drawing on battle experience from the Second World War, many experts claim that a 3-fold superiority in resources is typically needed for a successful conventional battle attack (see Kielmansegg 1977, pp.310-312). The claim has been made that such a rule holds more generally for a broader class of conflicts. Henry Kissinger (1960, p.809) claims that "conventional

[1] B could declare itself defeated, in which case it has also lost the war.

[2] Clausewitz (1832/1976, p.84) writes: "I am convinced that the superiority of defensive (if rightly understood) is very great, far greater than it appears at first sight."

warfare favors the defense", and reports that "even in World War II, the attacker generally required a superiority of three to one. The U.S. Minister of Defense James R. Schlesinger (1975, III-15) suggested a ratio of three to two. For further discussions see Kahn (1969, p.98n.), Canby (1975, p.12n.) and Stratmann (1981, p.52n.).

These assessments can be taken as evidence that there are at least some instances in which there is an advantage of defense, making it useful to analyse this case.[3] The fact that it takes only one party to start a contest together with an advantage of defense lead Clausewitz to the following resolution:

> Consequently, if the side favored by present conditions is not sufficiently strong to do without the added advantages of the defense, it will have to accept the prospect of acting under unfavorable conditions in the future. To fight a defensive battle under these less favorable conditions may still be better than to attack immediately or to make peace. (Clausewitz, 1832/1976, p.84)

Delay is an important empirical phenomenon also in other areas of economics and has been analyzed in various contexts. Uncertainty, revelation of information in the future, or asymmetric information is vital in most of these examples.[4] A type of one-sided delay that occurs in a full information

[3]In some other instances there might be an advantage of the attacker. This inverse asymmetry does not bring about delay: if there is an advantage of attack, the one who loses from delay can always attack and induce the showdown immediately.

[4]Waiting games are an example, particularly in the context of private provision of public goods if there is uncertainty about other contributors' types (Bliss and Nalebuff, 1984; Ghemawat and Nalebuff, 1985; Gradstein, 1992). Here all agents bear the cost of waiting, trying to shift the burden of contribution to others. Delay in situations when action reveals information that is also useful for others is related to this (e.g., Chamley and Gale, 1994; Gale, 1996; and Thimann and Thum, 1998). Delay has also been observed in bargaining games, for various reasons (Admati and Perry, 1987; Jehiel and Moldovanu, 1995a, 1995b; Sakovics, 1993). Schweizer (1989) and Spier (1994) consider pretrial bargaining and the choice between settling their dispute out of court and resorting to costly litigation. Further, in a situation in which superior information may arrive later, an agent may delay an irreversible investment choice in order to preserve an option value (McDonald and Siegel, 1986; and Pindyck, 1991, for an overview).

framework is analyzed by Hamilton and Slutsky (1990), explaining how sequential choice and Stackelberg leadership can evolve endogenously, and in the context of strategic trade, by Syropoulos (1994).

In the conflict we consider here, all agents are perfectly and completely informed and all agents delay. The stronger agent waits to become even stronger before trying to beat the weaker agent. The weaker agent does not like this, but the only way to accelerate the outcome is to assume a somewhat less advantageous role as attacker. While our analysis lends support to Clausewitz's claim, it also adds to his insights. First, it reveals that the size of the expected change in relative strength is crucial for whether there is delay. Delay does not occur if the change in relative strength is very large. A strong contestant may want to delay the showdown if his relative strength is further increasing in the future, but the contestant who is weaker than his competitor may force an early showdown if his relative strength is expected to deteriorate by a sufficiently large amount. Second, we consider the cost of future strength. Future military strength goes along with additional investment and, hence, additional cost. This additional cost of delay is neglected in Clausewitz's reasoning. It is an important force towards an early resolution of conflict. The cost is also interesting from a welfare point of view. Own cost of investment in future military strength is internalized in the decision to delay, the cost of the competitor is not.

2 A formal approach

Suppose there are two rulers A and B, each of which rules a fiefdom. The fiefdom represents a value to whomever rules it. For instance, this value could be the present value of future tax revenue that can be extracted. For simplicity, this revenue is some exogenously given amount T and the same in both fiefdoms.

There is conflict between the two rulers: before they arrive at the period in which these revenues accrue, they can try and defeat the other ruler, in which case the winner receives the incomes of both fiefdoms, and the loser receives zero. The interval in which war can take place consists of two periods. We call these periods $t = 0$ (early) and $t = 1$ (late). The rulers decide simultaneously in period 0 whether or not to attack. If at least one ruler decides to attack, a battle contest takes place in which one of the contestants is killed or finally defeated. If no attack takes place in period 0, then the rulers decide simultaneously in period 1 whether or not to attack. A contest in this period also leads to a final defeat of one of the contestants. Accordingly, the showdown takes place at 0, or at 1, or not at all, but not at both times.

The technology that determines the outcomes of military conflict is as follows. We distinguish between a situation in which both rulers decide to attack, and a situation in which only one ruler decides to attack, making the other ruler a defender. We will concentrate on the latter case, because in any pure strategy equilibrium at most one of the rulers will attack, and the description of what happens with simultaneous attack is needed only to make some out-of-equilibrium outcomes well defined. If both rulers decide to attack in the same period, a coin if tossed and determines who has the role of the attacker and who becomes the defender.

To describe the actual contest which may take place at 0 or at 1, we draw on the contest literature. Let ruler a be the attacker, and d the defender, and let $x_a \geq 0$ and $x_d \geq 0$ be the resources they spend in the contest. Then the attacker wins in the contest with a probability equal to $p(x_a, x_d)$ and the defender wins with the remaining probability $1 - p(x_a, x_d)$. We assume that there is an advantage of defense. That is, given the conflict resources x_A and x_B of contestants A and B, the contestant in the position of the attacker has a lower probability of winning than if he became the defender, i.e.,

$$p(x_A, x_B) < 1 - p(x_B, x_A). \qquad (1)$$

Further, the contest success function p is strictly increasing in its first and strictly decreasing in its second argument.

Let m_A and m_B be the rulers' conflict resources available in period 0, and let them be given exogenously and known by both rulers. It simplifies the analysis if we assume that conflict resources have no other use than in the contest, and hence, if there is a conflict, the contestants use all military resources they have[5] if one of them (or both) decide to attack at 0. In this case the military contest takes place and determines who wins and who loses. As the loser loses everything, there is no conflict at 1 and the winner receives the future returns on both territories, $2T$.

If no attack occurred at 0, the stock of military resources may change. Let n_A and n_B be the resources available in period 1. These are exogenous for most part of the analysis and also known to both rulers at the beginning of period 0. Making these resources available may involve some costs, which are incurred only if conflict does not take place in period 0 already. We denote these costs by $c_A(n_A) \geq 0$ and $c_B(n_B) \geq 0$, respectively.

The resources available for the military conflict develop in similar or opposite directions for A and B. Think of Hannibal trying to conquer the Roman Empire. As time moved on, Hannibal's army was weakened. He lost a major share of his war elephants, for instance. At the same time the Roman Empire could collect and redirect more resources into military uses. Similarly, a city which is under siege, may weaken while the attacker can collect and mobilize more troups from its own hinterland and increase the stock and the efficiency of his weapons, or the city may wait to receive support from allies, while the attacking army may suffer from disease and have used up all resources that can be gained from plundering the neighborhood.

Summarizing, the timing of actions and events is as follows. The values of m_A, m_B, n_A and n_B are known to both rulers at the beginnig of period 0 and exogenous. Each contestant decides whether to attack in period 0. If at least one attacks in period 0, the contest takes place, the winner is determined and the game ends. If none of them attacks, then n_A and n_B are generated at

[5] In a more general framework, part of the unused military resources could be converted back to consumer goods and be part of the payoff of the contest winner. However, while this introduces another interesting dimension, it generates a distinction between interior and corner solutions that distracts from the issue of delay which we concentrate on. Also, for a large parameter set in which T is sufficiently large compared to m_A and m_B, the contestants are budget constrained and use all resources m_A and m_B in the conflict anyway even if unused resources could be used for consumption.

the beginning of period 1 and the costs are $c_A(n_A)$ and $c_B(n_B)$, respectively. Then the rulers decide whether to attack in period 1.

Sufficient conditions for a delayed conflict are as follows.

Proposition 1 *Let* $n_A \geq n_B$ *be exogenously given. Sufficient conditions for a delayed conflict are*

$$p(n_A, n_B) > 1/2 \tag{2}$$

$$p(n_A, n_B) - p(m_A, m_B) > \frac{c_A(n_A)}{2T} \tag{3}$$

and

$$1 - p(n_A, n_B) - p(m_B, m_A) > \frac{c_B(n_B)}{2T}. \tag{4}$$

Proof: Let (2) be satisfied. Then A attacks in period 1 (if no contest has occurred in period 0.) Given that the conflict will occur in period 1, A prefers delay in period 0 if

$$2T p(m_A, m_B) < 2T p(n_A, n_B) - c_A(n_A)$$

and B prefers not to attack in period 0 if

$$2T p(m_B, m_A) < 2T(1 - p(n_A, n_B)) - c_B(n_B).$$

These conditions are equivalent to (3) and (4). Thus if (2) - (4) hold, there is an equilibrium with delayed conflict. ¤

Note that (3) and (4) can be fulfilled simultaneously even for $c_A(n_A) > 0$ and $c_B(n_B) > 0$ if there is an advantage of defense as described by (1), because this condition states that $p(m_A, m_B) + p(m_B, m_A) < 1$.

Condition (2) makes sure that A attacks B in period 1 if no attack took place in period 0. For $c_A(x) = c_B(x) = 0$ condition (3) states that A's win probability as an attacker in period 1 is higher than his win probability as an attacker in period 0, explaining why A prefers delay. If the cost in period 1 is positive, the increase in win probability in period 1 must compensate ruler A also for the cost of his additional investment in future military strength.

Condition (4) states that B is better off by waiting and becoming a defender in period 1 than by attacking in period 0. This condition makes the intuition of Clausewitz more precise. Even though B's conditions may worsen, B could be willing to accept a delay, because the only way to avoid a delay is to switch from the role of a defender to the role of an attacker, hence assuming the burden of attack. Also it reveals that evan a contestant who is weaker than his rival in period 0 may attack in period 0 if this contestant becomes even much weaker in period 1. We will illustrate this in an example below.

Conditions (3) and (4) also show that the costs of building up military power in period 1 further tighten the conditions for which delay occurs. This aspect is absent in Clausewitz's analysis. Delay causes an additional cost, and the contestants must be compensated for these additional cost in order to be willing to delay.

We can also consider welfare. In the absence of discounting and with an exogenous and symmetric valuation of winning the contest, delay reduces welfare by the cost $c(n_A) + c(n_B)$.

It is also interesting to discuss an endogenous choice of n_A and n_B between periods 0 and 1. In this case each contestant will consider the expected cost of continued conflict in the continuation equilibrium if no contest took place in period 0. This may but need not alter the outcome. For instance, let $c_A(n_A) = 0$ for $n_A \leq n_A^*$, $c_A(n_A) = \infty$ for $n_A > n_A^*$, and $c_B(n_B) = 0$ for $n_B \leq n_B^*$, and $c_B(n_B) = \infty$ for $n_B > \infty$, for some given n_A^* and n_B^*, with $p(n_A^*, n_B^*) > 1/2$. If no contest took place in period 0, both contestants will choose these thresholds n_A^* and n_B^* in the continuation equilibrium, A will attack in period 1. The equilibrium of the game with an endogenous choice of n_A and n_B is identical to the equilibrium of a game in which n_A^* and n_B^* are exogenously given.

The equilibrium outcome may differ if a large share of the total rents $2T$ is dissipated in period 1 if no contest took place in period 0, as this will induce the contestants to favor an early resolution of the conflict. Both contestants may want an early contest, but each may prefer the other contestant to attack. This can lead to a mixed strategy equilibrium in period 0 in which each contestant randomizes and chooses to attack in period 0 with some positive probability. Even in such mixed strategy equilibria delay occurs with a positive probability, but for a different reason than in the cases that are described by Proposition 1.

3 A parametric example

To illustrate how the equilibrium outcome depends on the asymmetry between attack and defense, we consider the following example. Suppose that

$$p(x_a, x_d) = \begin{cases} \frac{x_a}{x_a + kx_d} & \text{if} \quad \max\{x_a, x_d\} > 0 \\ 1/2 & \text{if} \quad x_a = x_d = 0 \end{cases} \tag{5}$$

and the defender wins with the remaining probability $1 - p$. This contest success probability is the same whether the attack occurs early or late. Here, $k \geq 1$ measures the defender's advantage: resources spent in defending are more effective in the contest than resources spent in attacking. To achieve the same win probability as a defender, an attacker must spend k times the effort of the defender.[6] For the case in which both rulers decide to attack, we continue to assume that the roles of attack and defense are assigned by the flip of a coin. If neither decides to attack at 0 or at 1, no contest takes place. Both rulers stay in power in this case and consume the incomes from their fiefdoms.

For simplicity we consider the case in which m_A, m_B, n_A and n_B are exogenous and $c_A(n_A) = c_B(n_B) = 0$.

Suppose no attack took place at 0, and resources in 1 are n_A and n_B. In this case the contestants are involved in the following game:

	attack	not attack
attack	$\frac{2k(n_A)^2+(1+k^2)n_A n_B}{2(n_A+kn_B)(kn_A+n_B)}2T, \frac{2k(n_B)^2+(1+k^2)n_A n_B}{2(n_A+kn_B)(kn_A+n_B)}2T$	$\frac{n_A}{n_A+kn_B}2T, \frac{kn_B}{n_A+kn_B}2T$
not attack	$\frac{kn_A}{kn_A+n_B}2T, \frac{n_B}{kn_A+n_B}2T$	T, T

Accordingly, if $n_A > kn_B$, A attacks and B does not attack, resulting in payoffs $G_A = \frac{n_A}{n_A+kn_B}2T$ and $G_B = \frac{kn_B}{n_A+kn_B}2T$. If $n_B > kn_A$, B attacks and

[6] This contest success function may look ad hoc at first sight. It is frequently used in various contexts, however. Skaperdas (1996) axiomatized the symmetric version ($k = 1$), and various micro-foundations that are related to innovation processes can be found in Mortensen (1982), Hirshleifer and Riley (1992), Baye and Hoppe (2001).

A does not attack, resulting in payoffs $G_A = \frac{kn_A}{kn_A+n_B}2T$ and $G_B = \frac{n_B}{kn_A+n_B}2T$. Finally, if $n_A < kn_B$ and $n_B < kn_A$, then no conflict occurs because neither party attacks. The reason is that the peaceful payoff even to the rival with the larger military resources is higher in the peaceful outcome than if this contestant assumes the role of attack.

Consider now stage one of the game: the decision whether to attack or not at time 0. Suppose the rivals both know how their own and their competitor's effective military resources develop. Then the following proposition holds:

Proposition 2 *(i) If* $\frac{m_B}{k} < m_A < km_B$ *and* $\frac{n_B}{k} < n_A < kn_B$ *then no conflict occurs. (ii) If* $m_i > km_j$ *or* $n_i > kn_j$*, then conflict occurs. (iii) Conflict is delayed to period* 1 *(even if* $m_i > km_j$*) if* $k^2 n_j m_i - m_j n_i > 0$*,* $n_i > kn_j$ *and* $\frac{n_i}{m_i} > \frac{n_j}{m_j}$.

Part (i) has a simple intuition. The condition $\frac{n_B}{k} < n_A < kn_B$ makes sure that it is disadvantageous for each ruler to attack at time 1 relative to no conflict. Knowing that there is no conflict at time 1, each of them prefers not to attack in period 0. Consider part (ii). If $n_i > kn_j$, and if there has been no conflict at 0, then i will attack at time 1. $m_i > km_j$ is also sufficient for conflict. Given this condition, conflict at 0 yields a higher outcome to i than T, the payoff which i receives if there is no conflict in each period. Hence, i need not attack at 0, but only if i plans to attack at time 1. Consider the three conditions determining delay in (iii). For delay it must hold that conflict is profitable for the attacker at time 1, which is the case for i by condition $n_i > kn_j$. Second, it must hold that, anticipating i's attack at time 1, neither i nor j want to attack at time 0. Condition $\frac{n_i}{m_i} > \frac{n_j}{m_j}$ makes sure that i gains from delay. However, if i gains from delay, this implies that j loses from the fact that i delays the conflict from time 0 to time 1. The alternative for j is to attack at time 0. However, $\frac{m_j}{m_j+km_i} < \frac{kn_j}{kn_j+n_i}$ makes sure that j prefers to be attacked at time 1 rather than being the attacker at time 0, and this condition reduces to $k^2 n_j m_i - m_j n_i > 0$. This concludes the proof. ¤

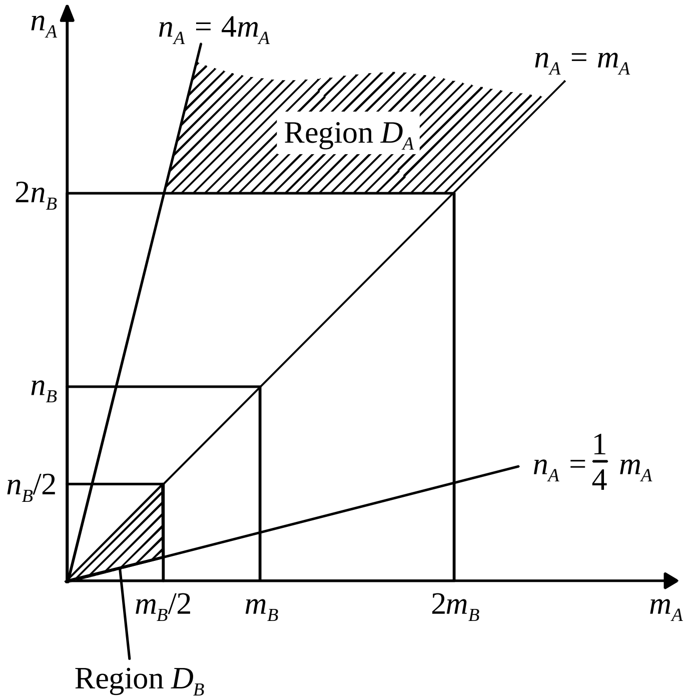

Figure 1: Equilibrium with and without delay

Figure 1 further illustrates the conditions in Proposition 1 for the case with $m_B = n_B$ and $k = 2$ that is, if only A's resources change. Let $\Omega \equiv \{(m_A, n_A) | m_A \geq 0, n_A \geq 0\}$. Conflict occurs everywhere outside the set $P \equiv \{(m_A, n_A) | (m_A, n_A) \in [\frac{m_B}{2}, 2m_B] \times [\frac{n_B}{2}, 2n_B]\}$. In all regions except $\Omega \setminus P$ a contest takes place in one of the periods. The contest is delayed to 1 in the regions D_A and D_B. In region D_A the ruler A delays his attack on B to period 1, even if $m_A > 2m_B$. In region D_B ruler B delays his attack on ruler A to period 1.

In all other regions $\Omega \setminus (P \cup D_A \cup D_B)$ the contest takes place in period 0. In some of these cases one ruler, say B, attacks the other ruler A in period 0 even though ruler B is less then twice as strong as A in that period (or A even stronger than B), because ruler B anticipates a very big increase in A's strength.

The comparative statics of delay can be analysed using Figure 1. An increase in k enlarges the region in which no contest at all takes place. This also shifts the lower boundary of region D_A upwards, and the upper boundary of region D_B downwards. However, the increase in k also shifts the constraint that limits region D_A on the left hand side further to the left, and the constraint that limits region D_B from the right hand side further to the right.

Figure 1 reveals that the asymmetry between attack and defense is crucial for delay as an equilibrium outcome. If the variable k that measures the efficiency advantage in defense approaches unity, both the regions in which no conflict occurs and the regions in which attack is delayed disappear. For $\mathsf{k} = 1$ conflict cannot be avoided and the ruler whose relative strength deteriorates between 0 and 1 attacks immediately.

4 Conclusions

This note reconsiders Clausewitz' conjecture about delay in conflict as an equilibrium outcome even under perfect information. If an attacker has a disadvantage in a conflict between two agents, both agents may want to delay the conflict; one agent gains from delaying his attack, the other agent loses from this delay, but prefers delay to assuming the role of an attacker. However, even the weaker agent may force an early showdown if this agent's strength weakens by too much. Delay is more likely to occur if the opportunity cost of future military resources is low, if the advantage of defense is large, and if the conditions for the contestant who is the weaker contestant in period 1 have deteriorated from period 0 to period 1, but not by too much.

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